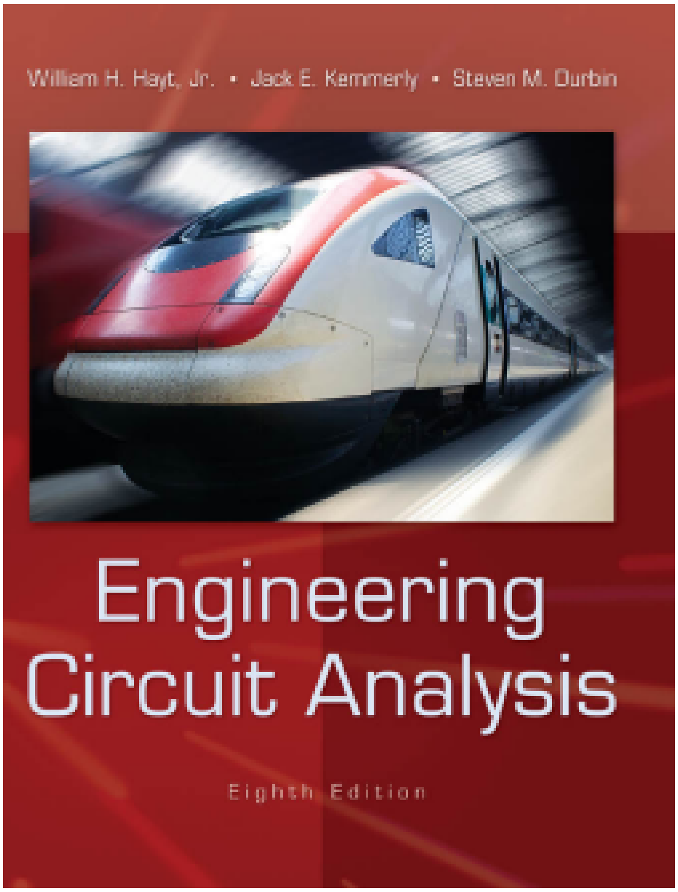


This page intentionally left blank

The Resistor Color Code

Band color Black Brown Red Orange Yellow Green Blue Violet Gray White **Numeric value** 0 123 4 56789

Multiplier

1st number

2nd number Tolerance band (e.g. gold = 5%
silver = 10%, none = 20%)

1. Write down the numeric value corresponding to the first band on the left.
2. Write down the numeric value corresponding to the second band from the left.
3. Write down the number of zeros indicated by the multiplier band, which represents a power of 10 (black = no extra zeros, brown = 1 zero, etc.). A gold multiplier band indicates that the decimal is shifted one place to the left; a silver multiplier band indicates that the decimal is shifted two places to the left.
4. The tolerance band represents the precision. So, for example, we would not be surprised to find a 100 5 percent tolerance resistor that measures anywhere in the range of 95 to 105 .

Example

Red Red Orange Gold = 22,000 or 22 × 10^3 = 22 k , 5% tolerance

Blue Gray Gold = 6.8 or 68 × 10^{-1} = 6.8 , 20% tolerance

Standard 5 Percent Tolerance Resistor Values

1.0 1.1 1.2 1.3 1.5 1.6 1.8 2.0 2.2 2.4 2.7 3.0 3.3 3.6 3.9 4.3 4.7 5.1 5.6 6.2 6.8 7.5 8.2 9.1
10. 11. 12. 13. 15. 16. 18. 20. 22. 24. 27. 30. 33. 36. 39. 43. 47. 51. 56. 62. 68. 75. 82. 91.
100 110 120 130 150 160 180 200 220 240 270 300 330 360 390 430 470 510 560 620 680 750 820 910
1.0 1.1 1.2 1.3 1.5 1.6 1.8 2.0 2.2 2.4 2.7 3.0 3.3 3.6 3.9 4.3 4.7 5.1 5.6 6.2 6.8 7.5 8.2 9.1 k
10. 11. 12. 13. 15. 16. 18. 20. 22. 24. 27. 30. 33. 36. 39. 43. 47. 51. 56. 62. 68. 75. 82. 91. k
100 110 120 130 150 160 180 200 220 240 270 300 330 360 390 430 470 510 560 620 680 750 820 910 k
1.0 1.1 1.2 1.3 1.5 1.6 1.8 2.0 2.2 2.4 2.7 3.0 3.3 3.6 3.9 4.3 4.7 5.1 5.6 6.2 6.8 7.5 8.2 9.1 M

TABLE 14.1 Laplace Transform Pairs

$f(t) = {}^{-1}\{F(s)\}$ $F(s) = \{f(t)\}$ $f(t) = {}^{-1}\{F(s)\}$ $F(s) = \{f(t)\}$ $\delta(t)$ 1 1

$\beta - \alpha$ $(e^{-\alpha t} - e^{-\beta t})u(t)$ 1

$(s + \alpha)(s + \beta)$

$u(t)$ $\frac{1}{s}$ $\sin \omega t\, u(t)$ ω

$s^2 + \omega^2$

$tu(t)$ $\frac{1}{s^2}$ $\cos \omega t\, u(t)$ s

t^{n-1}

$s^2 + \omega^2$

$(n-1)!\,u(t), n = 1, 2, \ldots$ $\frac{1}{s^n}$ $\sin(\omega t + \theta)\, u(t)$ $s \sin \theta + \omega \cos \theta$

$s^2 + \omega^2$

$e^{-\alpha t} u(t)$ 1

$s + \alpha$ $\cos(\omega t + \theta)\, u(t)$ $s \cos \theta - \omega \sin \theta$

$s^2 + \omega^2$

$te^{-\alpha t} u(t)$ 1

$(s + \alpha)^2$ $e^{-\alpha t} \sin \omega t\, u(t)$ ω

$(s + \alpha)^2 + \omega^2$

t^{n-1}

$(s + \alpha)^n$ $e^{-\alpha t} \cos \omega t\, u(t)$ $s + \alpha$

$(n-1)!\, e^{-\alpha t} u(t), n = 1, 2, \ldots$ 1

$(s + \alpha)^2 + \omega^2$

TABLE 6.1 Summary of Basic Op Amp Circuits

Name Circuit Schematic Input-Output Relation

i

R_f

R_1

−

+

+

i

+

− v_{in} v_{out}

−

R_f

R_1

−

+

+

v_{out}

+

− v_{in}

−

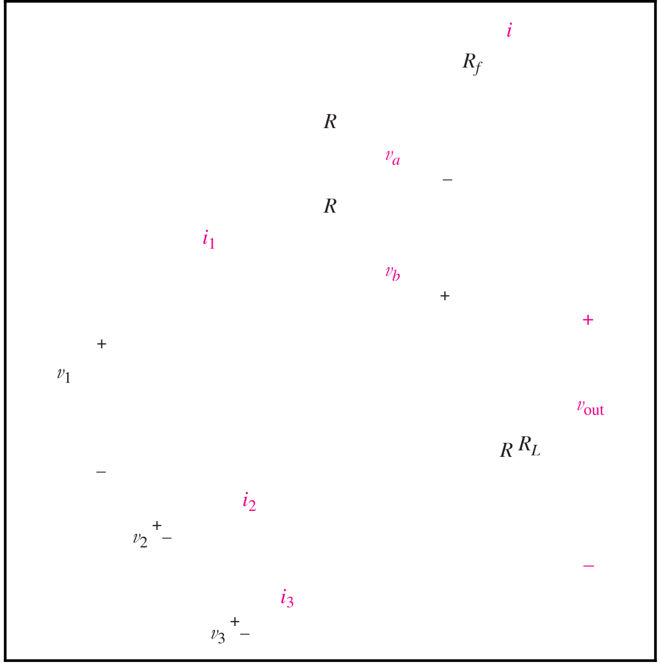
−

+

+

v_{in} + − v_{out}

−



Inverting Amplifier $v_{out} = -\frac{R_f}{R_1} v_{in}$

Noninverting Amplifier $v_{out} = \left(1 + \frac{R_f}{R_1}\right) v_{in}$

Voltage Follower $v_{out} = v_{in}$ (also known as a Unity Gain Amplifier)

Summing Amplifier $v_{out} = -\frac{R_f}{R}(v_1 + v_2 + v_3)$

Difference Amplifier $v_{out} = v_2 - v_1$

i_1

R R − +

+− v_2 + − i_2 v_a v_b +

R i

R v_1 R_L v_{out} −

ENGINEERING CIRCUIT ANALYSIS

This page intentionally left blank

ENGINEERING CIRCUIT ANALYSIS

EIGHTH EDITION

William H. Hayt, Jr. (deceased)
Purdue University

Jack E. Kemmerly (deceased)

California State University

Steven M. Durbin

University at Buffalo

The State University of New York

ENGINEERING CIRCUIT ANALYSIS, EIGHTH EDITION

Published by McGraw-Hill, a business unit of The McGraw-Hill Companies, Inc., 1221 Avenue of the Americas, New York, NY 10020.

This book is printed on acid-free paper.

1 2 3 4 5 6 7 8 9 0 DOW/DOW 1 0 9 8 7 6 5 4 3 2 1

ISBN 978-0-07-352957-8
MHID 0-07-352957-5

Vice President & Editor-in-Chief: *Marty Lange*
Vice President & Director of Specialized Publishing: *Janice M. Roerig-Blong*
Editorial Director: *Michael Lange*
Global Publisher: *Raghothaman Srinivasan*
Senior Marketing Manager: *Curt Reynolds*
Developmental Editor: *Darlene M. Schueller*
Lead Project Manager: *Jane Mohr*
Buyer: *Kara Kudronowicz*
Design Coordinator: *Brenda A. Rolwes*
Senior Photo Research Coordinator: *John C. Leland*
Senior Media Project Manager: *Tammy Juran*
Compositor: *MPS Limited, a Macmillan Company*
Typeface: *10/12 Times Roman*
Printer: *R. R. Donnelley*

Cover Image: © *Getty Images*
Cover Designer: *Studio Montage, St. Louis, Missouri*

The following photos are courtesy of Steve Durbin: Page 5, Fig. 2.22a, 2.24a–c, 5.34, 6.1a, 7.2a–c, 7.11a–b, 13.15, 17.29

Library of Congress Cataloging-in-Publication Data

Hayt, William Hart, 1920–1999
Engineering circuit analysis / William H. Hayt, Jr., Jack E. Kemmerly, Steven M. Durbin. — 8th ed.
p. cm.
Includes index.
ISBN 978-0-07-352957-8
1. Electric circuit analysis. 2. Electric network analysis. I. Kemmerly, Jack E. (Jack Ellsworth), 1924–1998 II. Durbin, Steven M. III. Title.

TK454.H4 2012
621.319'2—dc22 2011009912 www.mhhe.com

To Sean and Kristi. The best part of every day.

This page intentionally left blank

ABOUT THE AUTHORS

WILLIAM H. HAYT, Jr., received his B.S. and M.S. at Purdue University and his Ph.D. from the University of Illinois. After spending four years in industry, Professor Hayt joined the faculty of Purdue University, where he served as Professor and Head of the School of Electrical Engineering, and as Professor Emeritus after retiring in 1986. Besides *Engineering Circuit Analysis,* Professor Hayt authored three other texts, including *Engineering Electromagnetics,* now in its eighth edition with McGraw-Hill. Professor Hayt's professional society memberships included Eta Kappa Nu, Tau Beta Pi, Sigma Xi, Sigma Delta Chi, Fellow of IEEE, ASEE, and NAEB. While at Purdue, he received numerous teaching awards, including the univer sity's Best Teacher Award. He is also listed in Purdue's Book of Great Teachers, a permanent wall display in the Purdue Memorial Union, dedi cated on April 23, 1999. The book bears the names of the inaugural group of 225 faculty members, past and present, who have devoted their lives to excellence in teaching and scholarship. They were chosen by their students and their peers as Purdue's finest educators.

JACK E. KEMMERLY received his B.S. magna cum laude from The Catholic University of America, M.S. from University of Denver, and Ph.D. from Purdue University. Professor Kemmerly first taught at Purdue University and later worked as principal engineer at the Aeronutronic Division of Ford Motor Company. He then joined California State University, Fullerton, where he served as Professor, Chairman of the Faculty of Electrical Engi neering, Chairman of the Engineering Division, and Professor Emeritus. Professor Kemmerly's professional society memberships included Eta Kappa Nu, Tau Beta Pi, Sigma Xi, ASEE, and IEEE (Senior Member). His pursuits outside of academe included being an officer in the Little League and a scoutmaster in the Boy Scouts.

STEVEN M. DURBIN received the B.S., M.S. and Ph.D. degrees in Electrical Engineering from Purdue University,West Lafayette, Indiana. Subsequently, he was with the Department of Electrical Engineering at Florida State University and FloridaA&M University before joining the University of Canterbury, New Zealand,in2000.SinceAugust2010,hehasbeenwiththeUniversityatBuffalo, The State University of NewYork, where he holds a joint appointment between the Departments of Electrical Engineering and Physics. His teaching interests include circuits, electronics, electromagnetics, solid-state electronics and nanotechnology. His research interests are primarily concerned with the development of new semiconductor materials—in particular those based on ox ide and nitride compounds—as well as novel optoelectronic device structures. HeisafoundingprincipalinvestigatoroftheMacDiarmidInstituteforAdvanced Materials and Nanotechnology, a New Zealand National Centre of Research Excellence,andcoauthorofover100technicalpublications.Heisaseniormem ber of the IEEE, and a member of Eta Kappa Nu, the Electron Devices Society, the Materials Research Society, the AVS (formerly the American Vacuum Society),theAmericanPhysicalSociety,andtheRoyalSocietyofNewZealand.

This page intentionally left blank

This page intentionally left blank

CONTENTS

CHAPTER 9
THE *RLC* CIRCUIT 321

CHAPTER 10
SINUSOIDAL STEADY-STATE ANALYSIS 371

CHAPTER 11
AC CIRCUIT POWER ANALYSIS 421

CHAPTER 12
POLYPHASE CIRCUITS 457

CHAPTER 13
MAGNETICALLY COUPLED CIRCUITS 493

CHAPTER 14
COMPLEX FREQUENCY AND THE LAPLACE TRANSFORM 533

CHAPTER 15
CIRCUIT ANALYSIS IN THE s-DOMAIN 571

This page intentionally left blank

The target audience colors everything about a book, being a major fac tor in decisions big and small, particularly both the pace and the overall writing style. Consequently it is important to note that the au thors have made the conscious decision to write this book to the **student**, and not to the instructor. Our underlying philosophy is that reading the book should be enjoyable, despite the level of technical detail that it must incor porate. When we look back to the very first edition of *Engineering Circuit Analysis*, it's clear that it was developed specifically to be more of a con versation than a dry, dull discourse on a prescribed set of fundamental top ics. To keep it conversational, we've had to work hard at updating the book so that it continues to speak to the increasingly diverse group of students using it all over the world.

Although in many engineering programs the introductory circuits course is preceded or accompanied by an introductory physics course in which electricity and magnetism are introduced (typically from a fields perspec tive), this is not required to use this book. After finishing the course, many students find themselves truly amazed that such a broad set of analytical tools have been derived from **only three simple scientific laws**—Ohm's law and Kirchhoff's voltage and current laws. The first six chapters assume only a familiarity with algebra and simultaneous equations; subsequent chapters assume a first course in calculus (derivatives and integrals) is being taken in tandem. Beyond that, we have tried to incorporate sufficient details to allow the book to be read on its own.

So, what key features have been designed into this book with the student in mind? First, individual chapters are organized into relatively short sub sections, each having a single primary topic. The language has been up dated to remain informal and to flow smoothly. Color is used to highlight important information as opposed to merely improve the aesthetics of the page layout, and white space is provided for jotting down short notes and questions. New terms are defined as they are introduced, and examples are placed strategically to demonstrate not only basic concepts, but problem solving approaches as well. Practice problems relevant to the examples are placed in proximity so that students can try out the techniques for them selves before attempting the end-of-chapter exercises. The exercises repre sent a broad range of difficulties, generally ordered from simpler to more complex, and grouped according to the relevant section of each chapter. Answers to selected odd-numbered end-of-chapter exercises are posted on the book's website at www.mhhe.com/haytdurbin8e.

Engineering is an intensive subject to study, and students often find them selves faced with deadlines and serious workloads. This does not mean that textbooks have to be dry and pompous, however, or that coursework should never contain any element of fun. In fact, successfully solving a problem of ten *is* fun, and learning how to do that can be fun as well. Determining how to best accomplish this within the context of a textbook is an ongoing process. The authors have always relied on the often very candid feedback received from our own students at Purdue University; the California State University, Fullerton; Fort Lewis College in Durango, the joint engineering program at Florida A&M University and Florida State University, the Uni versity of Canterbury (New Zealand) and the University at Buffalo. We also rely on comments, corrections, and suggestions from instructors and students worldwide, and for this edition, consideration has been given to a new source of comments, namely, semianonymous postings on various websites.

The first edition of *Engineering Circuit Analysis* was written by Bill Hayt and Jack Kemmerly, two engineering professors who very much en joyed teaching, interacting with their students, and training generations of future engineers. It was well received due to its compact structure, “to the point” informal writing style, and logical organization. There is no timidity when it comes to presenting the theory underlying a specific topic, or pulling punches when developing mathematical expressions. Everything, however, was carefully designed to assist students in their learning, present things in a straightforward fashion, and leave theory for theory’s sake to other books. They clearly put a great deal of thought into writing the book, and their enthusiasm for the subject comes across to the reader.

KEY FEATURES OF THE EIGHTH EDITION

We have taken great care to retain key features from the seventh edition which were clearly working well. These include the general layout and se quence of chapters, the basic style of both the text and line drawings, the use of four-color printing where appropriate, numerous worked examples and related practice problems, and grouping of end-of-chapter exercises accord ing to section. Transformers continue to merit their own chapter, and com plex frequency is briefly introduced through a student-friendly extension of the phasor technique, instead of indirectly by merely stating the Laplace transform integral. We also have retained the use of icons, an idea first in troduced in the sixth edition:

Provides a heads-up to common mistakes;


Indicates a point that’s worth noting;


Denotes a design problem to which there is no unique answer;

Indicates a problem which requires computer-aided analysis.

The introduction of engineering-oriented analysis and design software in the book has been done with the mind-set that it should assist, not replace, the learning process. Consequently, the computer icon denotes problems that are typically phrased such that the software is used to *verify* answers, and not simply provide them. Both MATLAB® and PSpice® are used in this context.

SPECIFIC CHANGES FOR THE EIGHTH EDITION INCLUDE:

- A new section in Chapter 16 on the analysis and design of multistage Butterworth filters

- Over 1000 new and revised end-of-chapter exercises
- A new overarching philosophy on end-of-chapter exercises, with each section containing problems similar to those solved in worked examples and practice problems, before proceeding to more complex problems to test the reader's skills
- Introduction of Chapter-Integrating Exercises at the end of each chapter. For the convenience of instructors and students, end-of chapter exercises are grouped by section. To provide the opportunity for assigning exercises with less emphasis on an explicit solution method (for example, mesh or nodal analysis), as well as to give a broader perspective on key topics within each chapter, a select number of Chapter-Integrating Exercises appear at the end of each chapter.
- New photos, many in full color, to provide connection to the real world • Updated screen captures and text descriptions of computer-aided analysis software
- New worked examples and practice problems
- Updates to the Practical Application feature, introduced to help students connect material in each chapter to broader concepts in engineering. Topics include distortion in amplifiers, modeling automotive suspension systems, practical aspects of grounding, the relationship of poles to stability, resistivity, and the memristor, sometimes called "the missing element"
- Streamlining of text, especially in the worked examples, to get to the point faster
- Answers to selected odd-numbered end-of-chapter exercises are posted on the book's website at www.mhhe.com/haytdurbin8e.

I joined the book in 1999, and sadly never had the opportunity to speak to either Bill or Jack about the revision process, although I count myself lucky to have taken a circuits course from Bill Hayt while I was a student at Purdue. It is a distinct privilege to serve as a coauthor to *Engineering Circuit Analysis*, and in working on this book I give its fundamental philos ophy and target audience the highest priority. I greatly appreciate the many people who have already provided feedback—both positive and negative—on aspects of previous editions, and welcome others to do so as well, either through the publishers (McGraw-Hill Higher Education) or to me directly (durbin@ieee.org).

Of course, this project has been a team effort, as is the case with every modern textbook. In particular I would like to thank Raghu Srinivasan (Global Publisher), Peter Massar (Sponsoring Editor), Curt Reynolds (Mar keting Manager), Jane Mohr (Project Manager), Brittney-Corrigan McElroy (Project Manager), Brenda Rolwes (Designer), Tammy Juran (Media Project Manager), and most importantly, Developmental Editor Darlene Schueller, who helped me with many, many details, issues, deadlines,

and questions. She is absolutely the best, and I'm very grateful for all the support from the team at McGraw-Hill. I would also like to thank various McGraw-Hill representatives, especially Nazier Hassan, who dropped by whenever on campus to just say hello and ask how things were going. Spe cial thanks are also due to Catherine Shultz and Michael Hackett, former editors who continue to keep in contact. Cadence® and The MathWorks kindly provided assistance with software-aided analysis software, which was much appreciated. Several colleagues have generously supplied or helped with photographs and technical details, for which I'm very grateful: Prof. Masakazu Kobayashi of Waseda University; Dr. Wade Enright, Prof.

Pat Bodger, Prof. Rick Millane, Mr. Gary Turner, and Prof. Richard Blaikie of the University of Canterbury; and Prof. Reginald Perry and Prof. Jim Zheng of Florida A&M University and the Florida State University. For the eighth edition, the following individuals deserve acknowledgment and a debt of gratitude for taking the time to review various versions of the manuscript:

Chong Koo An, *The University of Ulsan*
Mark S. Andersland, *The University of Iowa*
Marc Cahay, *University of Cincinnati*
Claudio Canizares, *University of Waterloo*
Teerapon Dachokiatawan, *King Mongkut's University of Technology North Bangkok*
John Durkin, *The University of Akron*
Lauren M. Fuentes, *Durham College*
Lalit Goel, *Nanyang Technological University*
Rudy Hofer, *Conestoga College ITAL*
Mark Jerabek, *West Virginia University*
Michael Kelley, *Cornell University*
Hua Lee, *University of California, Santa Barbara*
Georges Livanos, *Humber College Institute of Technology*
Ahmad Nafisi, *Cal Poly State University*
Arnost Neugroschel, *University of Florida*
Pravin Patel, *Durham College*
Jamie Phillips, *The University of Michigan*
Daryl Reynolds, *West Virginia University*
G.V.K.R. Sastry, *Andhra University*
Michael Scordilis, *University of Miami*
Yu Sun, *University of Toronto, Canada*
Chanchana Tangwongsan, *Chulalongkorn University*
Edward Wheeler, *Rose-Hulman Institute of Technology*
Xiao-Bang Xu, *Clemson University*
Tianyu Yang, *Embry-Riddle Aeronautical University*
Zivan Zabar, *Polytechnic Institute of NYU*

I would also like to thank Susan Lord, *University of San Diego,* Archie L. Holmes, Jr., *University of Virginia,* Arnost Neugroschel, *University of Florida,* and Michael Scordilis, *University of Miami,* for their assistance in accuracy checking answers to selected end-of-chapter exercises.

Finally, I would like to briefly thank a number of other people who have contributed both directly and indirectly to the eighth edition. First and fore most, my wife, Kristi, and our son, Sean, for their patience, understanding, support, welcome distractions, and helpful advice. Throughout the day it has always been a pleasure to talk to friends and colleagues about what should be taught, how it should be taught, and how to measure learning. In particular, Martin Allen, Richard Blaikie, Alex Cartwright, Peter Cottrell, Wade Enright, Jeff Gray, Mike Hayes, Bill Kennedy, Susan Lord, Philippa Martin, Theresa Mayer, Chris McConville, Reginald Perry, Joan Redwing, Roger Reeves, Dick Schwartz, Leonard Tung, Jim Zheng, and many others have provided me with many useful insights, as has my father, Jesse Durbin, an electrical engineering graduate of the Indiana Institute of Technology.

Steven M. Durbin

Buffalo, New York

McGRAW-HILL DIGITAL OFFERINGS INCLUDE:

McGraw-Hill Connect™ Engineering

McGraw-Hill Connect Engineering is a web-based assignment and assessment platform that gives students the means to better connect with their coursework, with their instructors, and with the important concepts that they will need to know for success now and in the future. With Connect Engineering, instructors can deliver assignments, quizzes, and tests easily online. Students can practice important skills at their own pace and on their own schedule. Ask your McGraw-Hill representative for more details and check it out at www.mcgrawhillconnect.com/engineering.

Instructor support materials are available from the book's website at www.mhhe.com/haytdurbin8e. The materials include a password-protected solutions manual and image library. Instructors can also benefit from McGraw-Hill's Complete Online Solutions Manual Organization System (COSMOS). COSMOS enables instructors to generate a limitless supply of problem material for assignments, as well as transfer and integrate their own problems into the software. Contact your McGraw-Hill sales representative for additional information.

McGraw-Hill Create™

Craft your teaching resources to match the way you teach! With McGraw-Hill Create, www.mcgrawhillcreate.com, you can easily rearrange chapters, combine material from other content sources, and quickly upload content you have written such as your course syllabus or teaching notes. Find the content you need in Create by searching through thousands of leading McGraw-Hill textbooks. Arrange your book to fit your teaching style. Create even allows you to personalize your book's appearance by selecting the cover and adding your name, school, and course information. Order a Create book and you'll receive a complimentary print review copy in 3–5 business days or a complimentary electronic review copy (eComp) via email in minutes. Go to www.mcgrawhillcreate.com today and register to experience how McGraw-Hill Create empowers you to teach *your* students *your* way.

McGraw-Hill Higher Education and Blackboard® Have Teamed Up.

Blackboard, the web-based course management system, has partnered with McGraw-Hill to better allow students and faculty to use online materials and activities to complement face-to-face teaching. Blackboard features exciting social learning and teaching tools that foster more logical, visually

impactful, and active learning opportunities for students. You'll transform your closed-door classrooms into communities where students remain connected to their educational experience 24 hours a day.

This partnership allows you and your students access to McGraw-Hill's Connect™ and Create™ right from within your Blackboard course—all with one single sign-on.

Not only do you get single sign-on with Connect and Create, you also get deep integration of McGraw-Hill content and content engines right in Blackboard. Whether you're choosing a book for your course or building Connect assignments, all the tools you need are right where you want them—inside of Blackboard.

Gradebooks are now seamless. When a student completes an integrated Connect assignment, the grade for that assignment automatically (and in stantly) feeds your Blackboard grade center.

McGraw-Hill and Blackboard can now offer you easy access to industry leading technology and content, whether your campus hosts it or we do. Be sure to ask your local McGraw-Hill representative for details.

Electronic Textbook Option

This text is offered through CourseSmart for both instructors and students. CourseSmart is an online resource where students can purchase the complete text online at almost half the cost of a traditional text. Purchasing the eText book allows students to take advantage of CourseSmart's web tools for learn ing, which include full text search, notes and highlighting, and email tools for sharing notes between classmates. To learn more about CourseSmart options, contact your sales representative or visit www.CourseSmart.com.

This page intentionally left blank

Introduction

PREAMBLE

Although there are clear specialties within the field of engineering, all engineers share a considerable amount of common ground, particularly when it comes to problem solving. In fact, many prac ticing engineers find it is possible to work in a large variety of settings and even outside their traditional specialty, as their skill set is often transferrable to other environments. Today's engineering graduates are employed in a broad range of jobs, from design of individual components and systems, to assisting in solving socio economic problems such as air and water pollution, urban planning, communication, mass transportation, power generation and distribu tion, and efficient use and conservation of natural resources.

Circuit analysis has long been a traditional introduction to **the art of problem solving** from an engineering perspective, even for those whose interests lie outside electrical engineering. There are many reasons for this, but one of the best is that in today's world it's extremely unlikely for any engineer to encounter a system that does not in some way include electrical circuitry. As circuits be come smaller and require less power, and power sources become smaller and cheaper, embedded circuits are seemingly everywhere. Since most engineering situations require a team effort at some stage, having a working knowledge of circuit analysis therefore helps to provide everyone on a project with the background needed for effective communication.

Consequently, this book is not just about "circuit

analysis" from an engineering perspective, but is also about developing basic
problem-solving skills as they apply to situations an engineer is likely to encounter. As part of this, we also find that we're develop ing an intuitive understanding at a general level, and often we can

Linear versus Nonlinear Circuits

Four Main Categories of Circuit Analysis:

• DC Analysis

• Transient Analysis

• Sinusoidal Analysis

• Frequency Response

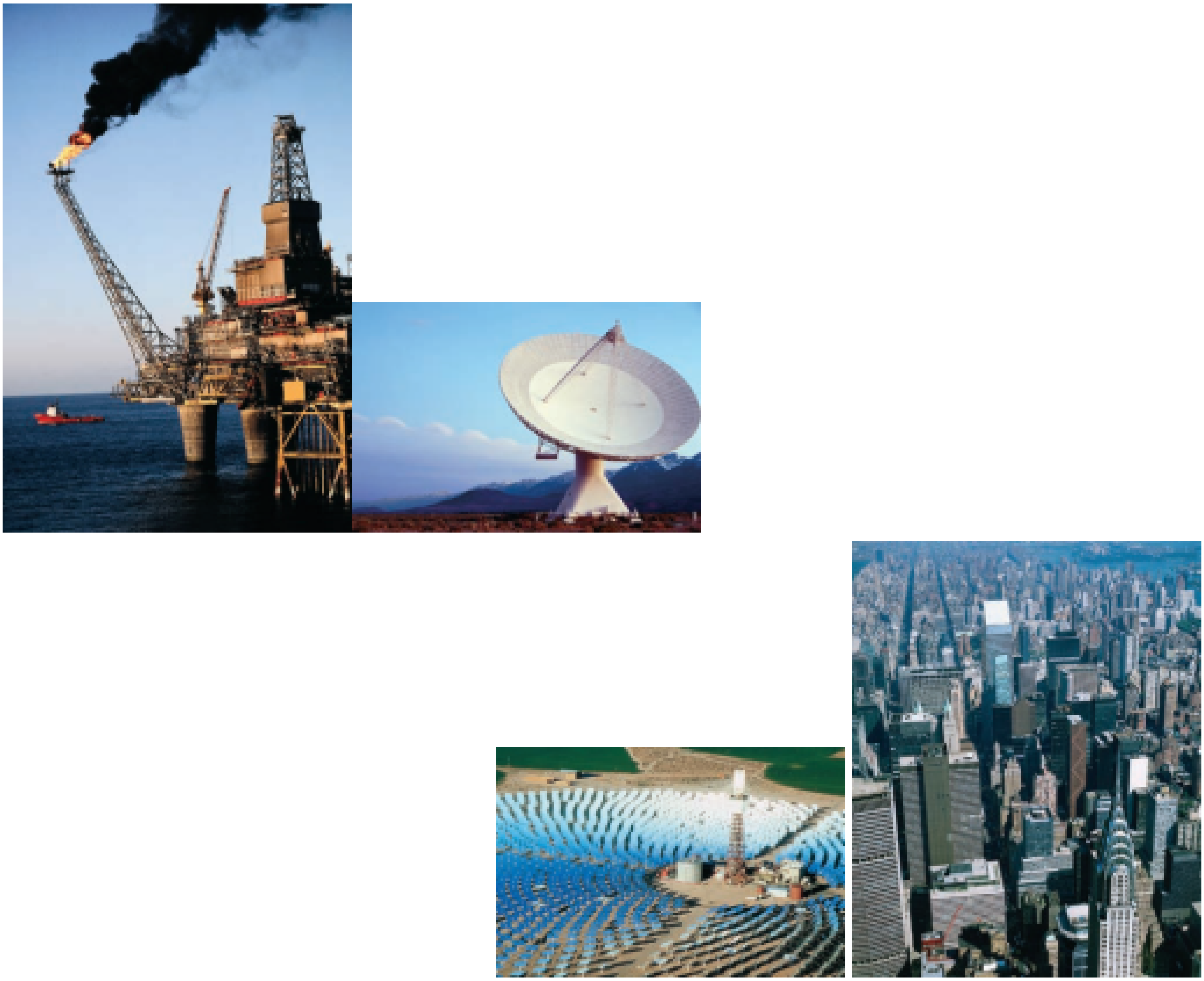
Circuit Analysis Beyond Circuits

Analysis and Design

Use of Engineering Software A Problem-Solving

Strategy

1



Not all electrical engineers routinely make use of circuit analysis, but they often bring to bear analytical and problem-solving skills learned early on in their careers. A circuit analysis course is one of the first exposures to such concepts. (*Solar Mirrors: © Corbis; Skyline: © Getty Images/PhotoLink; Oil Rig: © Getty Images; Dish: © Getty Images/J. Luke/PhotoLink*)

understand a complex system by its analogy to an electrical circuit. Before launching into all this, however, we'll begin with a quick preview of the topics found in the remainder of the book, pausing briefly to ponder the difference between analysis and design, and the evolving role computer tools play in modern engineering.

1.1 • OVERVIEW OF TEXT

The fundamental subject of this text is ***linear circuit analysis,*** which some times prompts a few readers to ask,

"Is there ever any *nonlinear* circuit analysis?"

Sure! We encounter nonlinear circuits every day: they capture and decode signals for our TVs and radios, perform calculations millions of times a second inside microprocessors, convert speech into electrical signals for transmission over phone lines, and execute many other functions outside our field of view. In designing, testing, and implementing such nonlinear circuits, detailed analysis is unavoidable.

“Then why study *linear* circuit analysis?” you might ask. An excellent question. The simple fact of the matter is that no physical system (including electrical circuits) is ever perfectly linear. Fortunately for us, however, a great many systems behave in a reasonably

Television sets include many nonlinear circuits. A great deal of them, however, can be understood and analyzed with the assistance of linear models. (© *Sony Electronics, Inc.*)

linear fashion over a limited range—allowing us to model them as linear systems if we keep the range limitations in mind.

For example, consider the common function

$$f(x) = e^x$$

A linear approximation to this function is

$$f(x) \approx 1 + x$$

Let’s test this out. Table 1.1 shows both the exact value and the approximate value of $f(x)$ for a range of x. Interestingly, the linear approximation is exceptionally accurate up to about $x = 0.1$, when the relative error is still less than 1%. Although many engineers are rather quick on a calculator, it’s hard to argue that any approach is faster than just adding 1.

TABLE **1.1** Comparison of a Linear Model for e^x to Exact Value

x	$f(x)$*	$1 + x$	Relative error**
0.0001	1.0001	1.0001	0.0000005%
0.001	1.0010	1.001	0.00005%
0.01	1.0101	1.01	0.005%
0.1	1.1052	1.1	0.5%
1.0	2.7183	2.0	26%

*Quoted to four significant figures.

**Relative error $100 \times \frac{e^x - (1 + x)}{e^x}$

Linear problems are inherently more easily solved than their nonlinear counterparts. For this reason, we often seek reasonably accurate linear approximations (or *models*) to physical situations. Furthermore, the linear models are more easily manipulated and understood—making design a more straightforward process.

The circuits we will encounter in subsequent chapters all represent linear approximations to physical electric circuits. Where appropriate, brief discussions of potential inaccuracies or limitations to these models are provided, but generally speaking we find them to be suitably accurate for most applications. When greater accuracy is required in practice, nonlinear models are employed, but with a considerable increase in solution complexity. A detailed discussion of what constitutes a *linear electric circuit* can be found in Chap. 2.

Linear circuit analysis can be separated into four broad categories: (1) *dc analysis,* where the energy sources do not change with time; (2) *transient analysis,* where things often change quickly; (3) *sinusoidal analysis,* which applies to both ac power and signals; and (4) *frequency* *response,* which is the most general of the four categories, but typically assumes something is changing with time. We begin our journey with the topic of resistive cir cuits, which may include simple examples such as a flashlight or a toaster. This provides us with a perfect opportunity to learn a number of very pow erful engineering circuit analysis techniques, such as *nodal analysis, mesh analysis, superposition, source transformation, Thévenin's theorem, Norton's*

Modern trains are powered by electric motors. Their electrical systems are best analyzed using ac or phasor analysis techniques. (*Used with permission. Image copyright © 2010 M. Kobayashi. All rights reserved.*)

theorem, and several methods for simplifying networks of components con nected in series or parallel. The single most redeeming feature of resistive circuits is that the time dependence of any quantity of interest does not affect our analysis procedure. In other words, if asked for an electrical quan tity of a resistive circuit at several specific instants in time, we do not need to analyze the circuit more than once. As a result, we will spend most of our effort early on considering only dc circuits—those circuits whose electrical parameters do not vary with time.

Although dc circuits such as flashlights or automotive rear window de foggers are undeniably important in everyday life, things are often much more interesting when something happens suddenly. In circuit analysis parlance, we refer to *transient analysis* as the suite of techniques used to study circuits which are suddenly energized or de-energized. To make such circuits interesting, we need to add elements that respond to the rate of change of electrical quantities, leading to circuit equations which include derivatives and integrals. Fortunately, we can obtain such equations using the simple techniques learned in the first part of our study.

Still, not all time-varying circuits are turned on and off suddenly. Air conditioners, fans, and fluorescent lights are only a few of the many exam ples we may see daily. In such situations, a calculus-based approach for every analysis can become tedious and time-consuming. Fortunately, there is a better alternative for situations where equipment has been allowed to run long enough for transient effects to die out, and this is commonly referred to as ac or sinusoidal analysis, or sometimes *phasor analysis*.

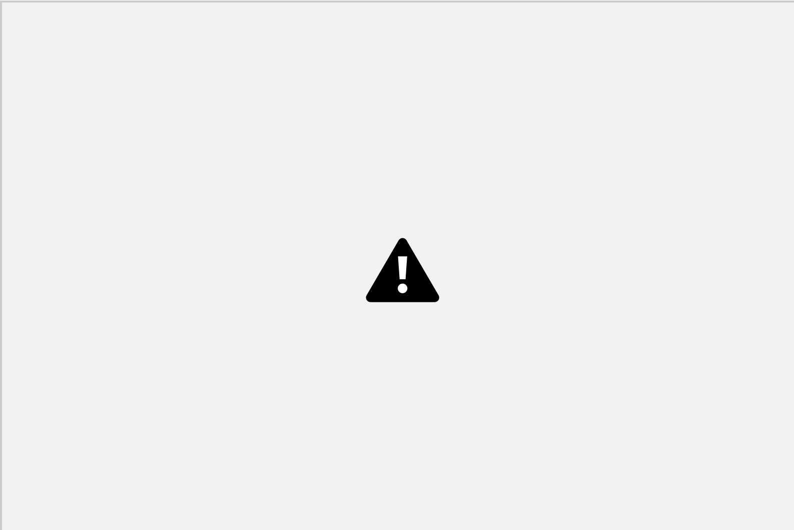
Frequency-dependent circuits lie at the heart of many electronic devices, and they can be a great deal of fun to design. (© *The McGraw-Hill Companies, Inc.*)

The final leg of our journey deals with a subject known as *frequency response*. Working directly with the differential equations obtained in time domain analysis helps us develop an intuitive understanding of the opera tion of circuits containing energy storage elements (e.g., capacitors and inductors). As we shall see, however, circuits with even a relatively small number of components can be somewhat onerous to analyze, and so much more straightforward methods have been developed. These methods, which include Laplace and Fourier analysis, allow us to transform differential equations into algebraic equations. Such methods also enable us to design circuits to respond in specific ways to particular frequencies. We make use of frequency-dependent circuits every day when we dial a telephone, select our favorite radio station, or connect to the Internet.

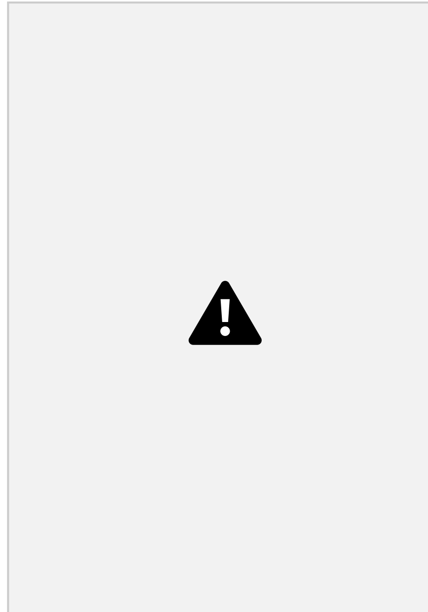
1.2 RELATIONSHIP OF CIRCUIT ANALYSIS TO ENGINEERING

Whether we intend to pursue further circuit analysis at the completion of this course or not, it is worth noting that there are several layers to the con cepts under study. Beyond the nuts and bolts of circuit analysis techniques lies the opportunity to develop a methodical approach to problem solving, the ability to determine the goal or goals of a particular problem, skill at collecting the information needed to effect a solution, and, perhaps equally importantly, opportunities for practice at verifying solution accuracy.

Students familiar with the study of other engineering topics such as fluid flow, automotive suspension systems, bridge design, supply chain manage ment, or process control will recognize the general form of many of the



A molecular beam epitaxy crystal growth facility. The equations governing its operation closely resemble those used to describe simple linear circuits.



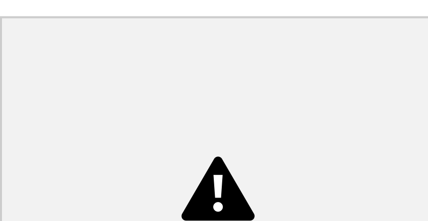
equations we develop to describe the behavior of various circuits. We simply
need to learn how to "translate" the relevant variables (for example, replacing
voltage with *force, charge* with *distance, resistance* with *friction coefficient,* etc.) to find that we already know how to work a new type of problem. Very often, if we have previous experience in solving a similar or related problem, our intuition can guide us through the solution of a totally new problem.

What we are about to learn regarding linear circuit analysis forms the basis for many subsequent electrical engineering courses. The study of elec tronics relies on the analysis of circuits with devices known as diodes and transistors, which are used to construct power supplies, amplifiers, and dig ital circuits. The skills which we will develop are typically applied in a rapid, methodical fashion by electronics engineers, who sometimes can analyze a complicated circuit without even reaching for a pencil! The time-domain and frequency-domain chapters of this text lead directly into discussions of signal processing, power transmission, control theory, and communications. We find that frequency-domain analysis in particular is an extremely powerful technique, easily applied to any physical system subjected to time-varying excitation, and particularly helpful in the design of filters.

1.3 ANALYSIS AND DESIGN

Engineers take a fundamental understanding of scientific principles, com bine this with practical knowledge often expressed in mathematical terms, and (frequently with considerable creativity) arrive at a solution to a given problem. ***Analysis*** is the process through which we determine the scope of a problem, obtain the information required to understand it, and compute the parameters of interest. ***Design*** is the process by which we synthesize something new as part of the solution to a problem. Generally speaking, there is an expectation that a problem requiring design will have no unique solution, whereas the analysis phase typically will. Thus, the last step in designing is always analyzing the result to see if it meets specifications.

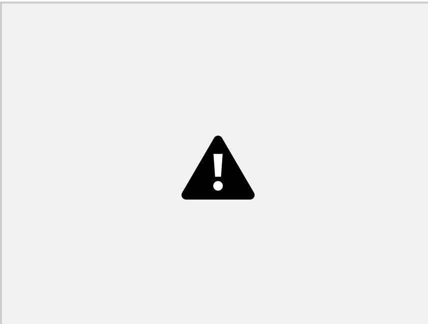
An example of a robotic manipulator. The feedback control system can be modeled using linear circuit elements to determine situations in which the operation may become unstable. (*NASA Marshall Space Flight Center.*)



This text is focused on developing our ability to analyze and solve problems because it is the starting point in every engineering situation. The philosophy of this book is that we need clear explanations, well-placed ex amples, and plenty of practice to develop such an ability. Therefore, elements of design are integrated into end-of-chapter problems and later chapters so as to be enjoyable rather than distracting.

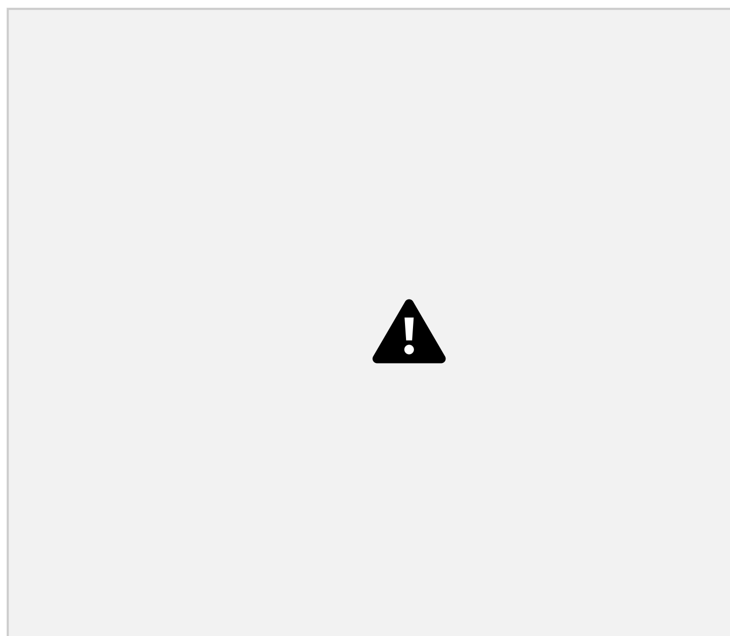
1.4 COMPUTER-AIDED ANALYSIS

Solving the types of equations that result from circuit analysis can often be come notably cumbersome for even moderately complex circuits. This of course introduces an increased probability that errors will be made, in addi tion to considerable time in performing the calculations. The desire to find a tool to help with this process actually predates electronic computers, with purely mechanical computers such as the Analytical Engine designed by Charles Babbage in the 1880s proposed as possible solutions. Perhaps the earliest successful electronic computer designed for solution of differential equations was the 1940s-era ENIAC, whose vacuum tubes filled a large room. With the advent of low-cost desktop computers, however, computer aided circuit analysis has developed into an invaluable everyday tool which has become an integral part of not only analysis but design as well.



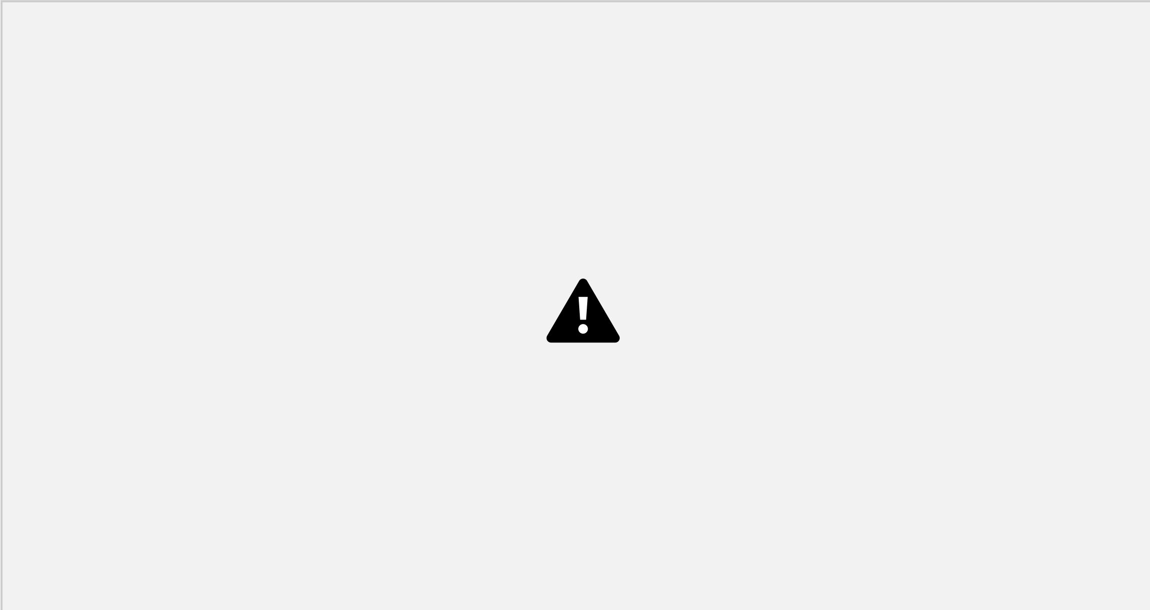
Two proposed designs for a next-generation space shuttle. Although both contain similar elements, each is unique. (*NASA Dryden Flight Research Center.*)

fashion transparent to the user. This allows the circuit to be drawn schematically on the screen, re duced automatically to the format required by an analysis program (such as SPICE, introduced in Chap. 4), and the resulting output smoothly trans ferred to a third program capable of plotting various electrical quantities of



Charles Babbage's "Difference Engine Number 2," as completed by the Science Museum (London) in 1991. (© *Science Museum/Science & Society Picture Library.*)

One of the most powerful aspects of computer-aided design is the rela tively recent integration of multiple programs in a



An amplifier circuit drawn using a commercial schematic capture software package.

interest that describe the operation of the circuit. Once the engineer is satis fied with the simulated performance of the design, the same software can generate the printed circuit board layout using geometrical parameters in the components library. This level of integration is continually increasing, to the point where soon an engineer will be able to draw a schematic, click a few buttons, and walk to the other side of the table to pick up a manufac tured version of the circuit, ready to test!

The reader should be wary, however, of one thing. Circuit analysis soft ware, although fun to use, is by no means a replacement for good old fashioned paper-and-pencil analysis. We need to have a solid understanding of how circuits work in order to develop an ability to design them. Simply going through the motions of running a particular software package is a little like playing the lottery: with user-generated entry errors, hidden default parame ters in the myriad of menu choices, and the occasional shortcoming of human written code, there is no substitute for having at least an approximate idea of the expected behavior of a circuit. Then, if the simulation result does not agree with expectations, we can find the error early, rather than after it's too late.

Still, computer-aided analysis is a powerful tool. It allows us to vary pa rameter values and evaluate the change in circuit performance, and to con sider several variations to a design in a straightforward manner. The result is a reduction of repetitive tasks, and more time to concentrate on engineer ing details.

1.5 SUCCESSFUL PROBLEM-SOLVING STRATEGIES

As the reader might have picked up, this book is just as much about problem solving as it is about circuit analysis. As a result, the expectation is that during your time as an engineering student, you are learning how to solve problems—so just at this moment, those skills are not yet fully developed. As you proceed

Read the problem statement slowly and carefully.

Collect the known information.

Identify the goal of the problem.

Devise a plan.

Construct an appropriate set of equations.

Determine
if additional information is required.

No

Attempt a solution.

Verify the
solution. Is it reasonable or expected?

Yes

End.

through your course of study, you will pick up techniques that work for you, and likely continue to do so as a practicing engineer. At this stage, then, we should spend a few moments discussing some basic points.

The first point is that by far, the most common difficulty encountered by engineering students is *not knowing how to start* a problem. This improves with experience, but early on that's of no help. The best advice we can give is to adopt a methodical approach, beginning with reading the problem statement slowly and carefully (and more than once, if needed). Since experience usually gives us some type of insight into how to deal with a specific problem, worked examples appear throughout the book. Rather than just read them, however, it might be helpful to work through them with a pencil and a piece of paper. Once we've read through the problem, and feel we might have some use ful experience, the next step is to identify the goal of the problem—perhaps to calculate a voltage or a power, or to select a component value. Knowing where we're going is a big help. The next step is to collect as much infor mation as we can, and to organize it somehow.

At this point *we're still not ready to reach for the calculator.* It's best first to devise a plan, perhaps based on experience, perhaps based simply on our intuition. Sometimes plans work, and sometimes they don't. Starting Yes

with our initial plan, it's time to construct an initial set of equations. If they appear complete, we can solve them. If not, we need to either locate more information, modify our plan, or both.

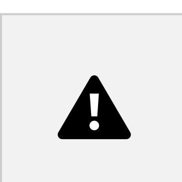
Once we have what appears to be a working solution, we should not stop, even if exhausted and ready for a break. **No engineering problem is solved unless the solution is tested somehow.** We might do this by per forming a computer simulation, or solving the problem a different way, or perhaps even just estimating what answer might be reasonable.

No

Since not everyone likes to read to learn, these steps are summarized in the adjacent flowchart. This is just one particular problem-solving strategy, and the reader of course should feel free to modify it as necessary. The real key, however, is to try and learn in a relaxed, low-stress environment free of distractions. Experience is the best teacher, and learning from our own mis takes will always be part of the process of becoming a skilled engineer.

READING FURTHER

This relatively inexpensive, best-selling book teaches the reader how to develop winning strategies in the face of seemingly impossible problems: G. Polya, *How to Solve It.* Princeton, N.J.: Princeton University Press, 1971.



Basic Components

and Electric Circuits

INTRODUCTION

In conducting circuit analysis, we often find ourselves seeking spe cific *currents, voltages,* or *powers,* so here we begin with a brief de scription of these quantities. In terms of components that can be used to build electrical circuits, we have quite a few from which to choose. We initially focus on the *resistor,* a simple passive compo nent, and a range of idealized active sources of voltage and current. As we move forward, new components will be added to the inven tory to allow more complex (and useful) circuits to be considered.

A quick word of advice before we begin: Pay close attention to the role of "**+**" and "**−**" signs when labeling voltages, and the sig nificance of the arrow in defining current; they often make the difference between wrong and right answers.

2.1 UNITS AND SCALES

In order to state the value of some measurable quantity, we must give both a *number* and a *unit,* such as "3 meters." Fortunately, we all use the same number system. This is not true for units, and a lit tle time must be spent in becoming familiar with a suitable system. We must agree on a standard unit and be assured of its permanence and its general acceptability. The standard unit of length, for exam ple, should not be defined in terms of the distance between two marks on a certain rubber band; this is not permanent, and further more everybody else is using another standard.

The most frequently used system of units is the one adopted by the National Bureau of Standards in 1964; it is used by all major professional engineering societies and is the language in which to day's textbooks are written. This is the International System of Units (abbreviated ***SI*** in all languages), adopted by the General

Basic Electrical Quantities and Associated Units:
Charge, Current, Voltage, and Power

Current Direction and
Voltage Polarity

The Passive Sign Convention for Calculating Power

Ideal Voltage and Current Sources

Dependent Sources

Conference on Weights and Measures in 1960. Modified several times since, the SI is built upon seven basic units: the *meter, kilogram, second, ampere, kelvin, mole,* and *candela* (see Table 2.1). This is a "metric system," some form of which is now in common use in most countries of the world, although it is not yet widely used in the United States. Units for other quan tities such as volume, force, energy, etc., are derived from these seven base units.

TABLE 2.1 SI Base Units

Base Quantity	Name	Symbol
length	meter	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

The fundamental unit of work or energy is the ***joule*** (J). One joule (a kg m^2 s^{-2} in SI base units) is equivalent to 0.7376 foot pound-force (ft · lbf). Other energy units include the calorie (cal), equal to 4.187 J; the British thermal unit (Btu), which is 1055 J; and the kilowatthour (kWh), equal to 3.6×10^6 J. Power is defined as the *rate* at which work is done or energy is expended. The fundamental unit of power is the ***watt*** (W), defined as 1 J/s. One watt is equivalent to 0.7376 ft · lbf/s or, equivalently, 1/745.7 horsepower (hp).

There is some inconsistency regarding whether units named after a person should be capitalized. Here, we will adopt the most contemporary convention,[1,2] where such units are written out in lowercase (e.g., watt, joule), but abbreviated with an uppercase symbol (e.g., W, J).

(1) H. Barrell, *Nature* 220, 1968, p. 651.
(2) V. N. Krutikov, T. K. Kanishcheva, S. A. Kononogov, L. K. Isaev, and N. I. Khanov, *Measurement Techniques* 51, 2008, p. 1045.

The "calorie" used with food, drink, and exercise is really a kilocalorie, 4.187 J.

The SI uses the decimal system to relate larger and smaller units to the basic unit, and employs prefixes to signify the various powers of 10. A list of prefixes and their symbols is given in Table 2.2; the ones most commonly encountered in engineering are highlighted.

TABLE 2.2 SI Prefixes

Factor	Name	Symbol	Factor	Name	Symbol
10^{-24}	yocto	y	10^{24}	yotta	Y
10^{-21}	zepto	z	10^{21}	zetta	Z
10^{-18}	atto	a	10^{18}	exa	E
10^{-15}	femto	f	10^{15}	peta	P
10^{-12}	pico	p	10^{12}	tera	T
10^{-9}	nano	n	10^{9}	giga	G
10^{-6}	micro	μ	10^{6}	mega	M
10^{-3}	milli	m	10^{3}	kilo	k
10^{-2}	centi	c	10^{2}	hecto	h
10^{-1}	deci	d	10^{1}	deka	da

These prefixes are worth memorizing, for they will appear often both in this text and in other technical work. Combinations of several prefixes, such as the millimicrosecond, are unacceptable. It is worth noting that in terms of distance, it is common to see "micron (μm)" as opposed to "microme ter," and often the angstrom (Å) is used for 10^{-10} meter. Also, in circuit analysis and engineering in general, it is fairly common to see numbers ex pressed in what are frequently termed "engineering units." In engineering notation, a quantity is represented by a number between 1 and 999 and an appropriate metric unit using a power divisible by 3. So, for example, it is preferable to express the quantity 0.048 W as 48 mW, instead of 4.8 cW, 4.8×10^{-2} W, or 48,000 μW.

PRACTICE •

2.1 A krypton fluoride laser emits light at a wavelength of 248 nm. This is the same as: (*a*) 0.0248 mm; (*b*) 2.48 μm; (*c*) 0.248 μm; (*d*) 24,800 Å.

2.2 A single logic gate in a prototype integrated circuit is found to be capable of switching from the "on" state to the "off" state in 12 ps. This corresponds to: (*a*) 1.2 ns; (*b*) 120 ns; (*c*) 1200 ns; (*d*) 12,000 ns. 2.3 A typical incandescent reading lamp runs at 60 W. If it is left on constantly, how much energy (J) is consumed per day, and what is the weekly cost if energy is charged at a rate of 12.5 cents per kilowatthour?

Ans: 2.1 (c); 2.2 (d); 2.3 5.18 MJ, $1.26.

2.2 • CHARGE, CURRENT, VOLTAGE, AND POWER

Charge

One of the most fundamental concepts in electric circuit analysis is that of charge conservation. We know from basic physics that there are two types

As seen in Table 2.1, the base units of the SI are not derived from fundamental physical quantities. Instead,

of charge: positive (corresponding to a proton) and negative (corresponding to an electron). For the most part, this text is concerned with circuits in which only electron flow is relevant. There are many devices (such as bat teries, diodes, and transistors) in which positive charge motion is important to understanding internal operation, but external to the device we typically concentrate on the electrons which flow through the connecting wires. Although we continuously transfer charges between different parts of a cir cuit, we do nothing to change the total amount of charge. In other words, we neither create nor destroy electrons (or protons) when running electric circuits.[1] Charge in motion represents a *current*.

In the SI system, the fundamental unit of charge is the ***coulomb*** (C). It is defined in terms of the ***ampere*** by counting the total charge that passes through an arbitrary cross section of a wire during an interval of one second; one coulomb is measured each second for a wire carrying a current of 1 ampere (Fig. 2.1). In this system of units, a single electron has a charge of -1.602×10^{-19} C and a single proton has a charge of $+1.602 \times 10^{-19}$ C.

(1) Although the occasional appearance of smoke may seem to suggest otherwise. . .

they represent historically agreed upon measurements, leading to definitions which occasionally seem backward. For example, it would make more sense physically to define the ampere based on electronic charge.

Individual charges

Cross section

Direction of
charge motion

■ **FIGURE 2.1** The definition of current illustrated using current flowing through a wire; 1 ampere corresponds to 1 coulomb of charge passing through the arbitrarily chosen cross section in 1 second.

A quantity of charge that does not change with time is typically repre sented by Q. The instantaneous amount of charge (which may or may not be time-invariant) is commonly represented by $q(t)$, or simply q. This conven tion is used throughout the remainder of the text: capital letters are reserved for constant (time-invariant) quantities, whereas lowercase letters represent the more general case. Thus, a constant charge may be represented by *either* Q or q, but an amount of charge that changes over time *must* be represented by the lowercase letter q.

Current

The idea of "transfer of charge" or "charge in motion" is of vital importance to us in studying electric circuits because, in moving a charge from place to place, we may also transfer energy from one point to another. The familiar cross-country power-transmission line is a practical example of a device that transfers energy. Of equal importance is the possibility of varying the rate at which the charge is transferred in order to communicate or transfer information. This process is the basis of communication systems such as radio, television, and telemetry.

The current present in a discrete path, such as a metallic wire, has both a *numerical value* and a *direction* associated with it; it is a measure of the rate at which charge is moving past a given reference point in a specified direction.

Once we have specified a reference direction, we may then let $q(t)$ be the total charge that has passed the reference point since an arbitrary time $t = 0$, moving in the defined direction. A contribution to this total charge will be negative if negative charge is moving in the reference direction, or if posi tive charge is moving in the opposite direction. As an example, Fig. 2.2 shows a history of the total charge $q(t)$ that has passed a given reference point in a wire (such as the one shown in Fig. 2.1).

q(t) (C)
6
5
4
3
2
1
0
−1 −2
12345678
t(s)

■ **FIGURE 2.2** A graph of the instantaneous value of the total charge $q(t)$ that has passed a given reference point since t 0.

We define the current at a specific point and flowing in a specified direc tion as the instantaneous rate at which net positive charge is moving past that point in the specified direction. This, unfortunately, is the historical de finition, which came into popular use before it was appreciated that current in wires is actually due to negative, not positive, charge motion. Current is symbolized by I or i, and so

$$i = \frac{dq}{dt} \quad [1]$$

The unit of current is the ampere (A), named afterA. M.Ampère, a French physicist. It is commonly abbreviated as an "amp," although this is unofficial and somewhat informal. One ampere equals 1 coulomb per second.

Using Eq. [1], we compute the instantaneous current and obtain Fig. 2.3. The use of the lowercase letter i is again to be associated with an instantaneous value; an uppercase I would denote a constant (i.e., time-invariant) quantity.

i(t) (A)
1.5
1
0.5
0
–0.5
12345678
t(s)

The charge transferred between time t_0 and t may be expressed as a

–1

–1.5

–2

definite integral: $q(t)$ $q(t_0)$ t t_0

$dq =$ $i\,dt$

The total charge transferred over all time is thus given by

in Fig. 2.2.

t t_0 $i\,dt + q(t_0)$ [2]

■ **FIGURE 2.3** The instantaneous current i dq/dt, where q is given

$q(t) =$

Several different types of current are illustrated in Fig. 2.4. A current that is constant in time is termed a direct current, or simply dc, and is shown by Fig. 2.4*a*. We will find many practical examples of currents that vary si nusoidally with time (Fig. 2.4*b*); currents of this form are present in normal household circuits. Such a current is often referred to as alternating current, or ac. Exponential currents and damped sinusoidal currents (Fig. 2.4*c* and *d*) will also be encountered later.

We create a graphical symbol for current by placing an arrow next to the conductor. Thus, in Fig. 2.5*a* the direction of the arrow and the value 3 A in dicate either that a net positive charge of 3 C/s is moving to the right or that a net negative charge of −3 C/s is moving to the left each second. In Fig. 2.5*b* there are again two possibilities: either −3 A is flowing to the left or +3 A is flowing to the right. All four statements and both figures represent currents that are equivalent in their electrical effects, and we say that they are equal. A nonelectrical analogy that may be easier to visualize is to think in terms of a personal savings account: e.g., a deposit can be viewed as either a *negative* cash flow *out of* your account or a *positive* flow *into* your account.

t

(*c*)

i

t

i

(*b*)

i

t

t

(*a*)

i

(*d*)

It is convenient to think of current as the motion of positive charge, even though it is known that current flow in metallic conductors results from electron motion. In ionized gases, in electrolytic solutions, and in some semiconductor materials, however, positive charges in motion consti tute part or all of the current. Thus, any definition of current can agree with the physical nature of conduction only part of the time. The definition and symbolism we have adopted are standard.

$i_1(t)$

■ **FIGURE 2.4** Several types of current: (*a*) Direct current (dc). (*b*) Sinusoidal current (ac).
(*c*) Exponential current. (*d*) Damped sinusoidal current.

It is essential that we realize that the current arrow does not indicate the "actual" direction of current flow but is simply part of a convention that allows us to talk about "the current in the wire" in an unambiguous manner. The arrow is a fundamental part of the definition of a current! Thus, to talk about the value of a current $i_1(t)$ without specifying the arrow is to discuss an undefined entity. For example, Fig. 2.6*a* and *b* are meaningless represen tations of $i_1(t)$, whereas Fig. 2.6*c* is complete.

3 A

–3 A

(*b*)

(*a*)

$i_1(t)$ ■ **FIGURE 2.5** Two methods of representation for the exact same current.

(*a*) (*b*) (*c*)

■ **FIGURE 2.6** (*a*, *b*) Incomplete, improper, and incorrect definitions of a current. (*c*) The correct definition of $i_1(t)$.

PRACTICE •

2.4 In the wire of Fig. 2.7, electrons are moving *left* to *right* to create a current of 1 mA. Determine I_1 and I_2.

I_1

I_2

■ **FIGURE 2.7**

Ans: $I_1 = -1$ mA; $I_2 = +1$ mA.

Voltage

A

B

■ **FIGURE 2.8** A general two-terminal circuit element.

We must now begin to refer to a circuit element, something best defined in general terms to begin with. Such electrical devices as fuses, light bulbs, re sistors, batteries, capacitors, generators, and spark coils can be represented by combinations of simple circuit elements. We begin by showing a very general circuit element as a shapeless object possessing two terminals at which connections to other elements may be made (Fig. 2.8).

There are two paths by which current may enter or leave the element. In subsequent discussions we will define particular circuit elements by describ ing the electrical characteristics that may be observed at their terminals. In Fig. 2.8, let us suppose that a dc current is sent into terminal *A*, through the general element, and back out of terminal *B*. Let us also assume that pushing charge through the element requires an expenditure of energy. We then say that an electrical voltage (or a *potential difference*) exists be tween the two terminals, or that there is a voltage "across" the element. Thus, the voltage across a terminal pair is a measure of the work required to move charge through the element. The unit of voltage is the volt,[2] and 1 volt is the same as 1 J/C. Voltage is represented by V or *v*.

A

+

$v = -5$ V −

B

(*a*)

A

+

A

−

$v = 5$ V +

B

(*b*)

$v = 5$ V

−

B

(*c*)

$v = -5$ V +

B

(*d*)

A

−

■ **FIGURE 2.9** (*a*, *b*) Terminal *B* is 5 V positive with respect to terminal *A*; (*c*, *d*) terminal *A* is 5 V positive with respect to terminal *B*.

Avoltage can exist between a pair of electrical terminals whether a current is flowing or not. An automobile battery, for example, has a voltage of 12 V across its terminals even if nothing whatsoever is connected to the terminals. According to the principle of conservation of energy, the energy that is expended in forcing charge through the element must appear somewhere else. When we later meet specific circuit elements, we will note whether that energy is stored in some form that is readily available as electric energy or whether it changes irreversibly into heat, acoustic energy, or some other nonelectrical form.

We must now establish a convention by which we can distinguish be tween energy supplied *to* an element and energy that is supplied *by* the element itself. We do this by our choice of sign for the voltage of terminal *A* with respect to terminal *B*. If a positive current is entering terminal *A* of the element and an external source must expend energy to establish this cur rent, then terminal *A* is positive with respect to terminal *B*. (Alternatively, we may say that terminal *B* is negative with respect to terminal *A*.)

The sense of the voltage is indicated by a plus-minus pair of algebraic signs. In Fig. 2.9*a*, for example, the placement of the + sign at terminal *A* indicates that terminal *A* is *v* volts positive with respect to terminal *B*. If we later find that *v* happens to have a numerical value of −5 V,

+

$v_1(t)$

−

(*a*)

+

then we may say

(*b*)

either that *A* is −5 V positive with respect to *B* or that *B* is 5 V positive with respect to *A*. Other cases are shown in Fig. 2.9*b*, *c*, and *d*.

Just as we noted in our definition of current, it is essential to realize that

$v_1(t)$

−

(*c*)

■ **FIGURE 2.10** (*a, b*) These are inadequate definitions of a voltage. (*c*) A correct definition includes both a symbol for the variable and a plus-minus symbol pair.

the plus-minus pair of algebraic signs does not indicate the "actual" polarity of the voltage but is simply part of a convention that enables us to talk unam biguously about "the voltage across the terminal pair." *The definition of any voltage must include a plus-minus sign pair*! Using a quantity $v_1(t)$ without specifying the location of the plus-minus sign pair is using an undefined term. Figure 2.10*a* and *b* do *not* serve as definitions of $v_1(t)$; Fig. 2.10*c* does.

(2) We are probably fortunate that the full name of the 18th century Italian physicist, *Alessandro Giuseppe Antonio Anastasio Volta,* is not used for our unit of potential difference!

PRACTICE •

2.5 For the element in Fig. 2.11, v_1 = 17 V. Determine v_2.

+ v_1 −   − v_2 +

Ans: v_2 = −17 V.

■ **FIGURE 2.11**

Power

We have already defined power, and we will represent it by *P* or *p*. If one joule of energy is expended in transferring one coulomb of charge through the device in one second, then the rate of energy transfer is one watt. The absorbed power must be proportional both to the number of coulombs trans ferred per second (current) and to the energy needed to transfer one coulomb through the element (voltage). Thus,

i

+

$$p = vi \quad [3]$$

v

−

Dimensionally, the right side of this equation is the product of joules per coulomb and coulombs per second, which produces the expected dimension of joules per second, or watts. The conventions for current, voltage, and power are shown in Fig. 2.12.

We now have an expression for the power being absorbed by a circuit element in terms of a voltage across it and current through it. Voltage was defined in terms of an energy expenditure, and power is the rate at which en ergy is expended. However, no statement can be made concerning energy transfer in any of the four cases shown in Fig. 2.9, for example, until the direction of the current is specified. Let us imagine that a current arrow is placed alongside each upper lead, directed to the right, and labeled "+2 A." First, consider the case shown in Fig. 2.9*c*. Terminal *A* is 5 V positive with respect to terminal *B*, which means that 5 J of energy is required to move each coulomb of positive charge into terminal *A*, through the object, and out terminal *B*. Since we are injecting +2 A (a current of 2 coulombs of positive charge per second) into terminal *A*, we are doing (5 J/C) × (2 C/s)   10 J of work per second on the object. In other words, the object is absorbing 10 W of power from whatever is injecting the current.

We know from an earlier discussion that there is no difference between Fig. 2.9*c* and Fig. 2.9*d*, so we expect the object depicted in Fig. 2.9*d* to also be absorbing 10 W. We can check this easily enough: we are injecting +2 A into terminal *A* of the object, so +2 A flows out of terminal *B*. Another way of saying this is that we are injecting −2 A of current into terminal *B*. It takes −5 J/C to move charge from terminal B to terminal A, so the object is absorbing (−5 J/C) × (−2 C/s)   +10 W as expected. The only difficulty in describing this particular case is keeping the minus signs straight, but with a bit of care we see the correct answer can be obtained regardless of our choice of positive reference terminal (terminal A in Fig. 2.9*c*, and terminal B in Fig. 2.9*d*).

■ **FIGURE 2.12** The power absorbed by the element is given by the product p vi. Alternatively, we can say that the element generates or supplies a power $-vi$.

Now let's look at the situation depicted in Fig. 2.9*a*, again with +2 A injected into terminal A. Since it takes −5 J/C to move charge from terminal A to terminal B, the object is absorbing (−5 J/C) × (2 C/s)   −10 W. What does this mean? How can anything absorb ***negative*** power? If we think about this in terms of energy transfer, −10 J is transferred to the object each second through the 2 A current flowing into terminal A. The object is actually losing energy—at a rate of 10 J/s. In other words, it is supplying 10 J/s (i.e., 10 W) to some other object not shown in the figure. Negative *absorbed* power, then, is equivalent to positive *supplied* power.

Let's recap. Figure 2.12 shows that if one terminal of the element is v volts positive with respect to the other terminal, and if a current i is entering the element through that terminal, then a power $p = vi$ is being *absorbed* by the element; it is also correct to say that a power $p = vi$ is being *delivered* to the element. When the current arrow is directed into the element at the plus-marked terminal, we satisfy the ***passive sign convention.*** This convention should be studied carefully, understood, and memorized. In other words, it says that if the current arrow and the voltage polarity signs are placed such that the current enters that end of the element marked with the positive sign, then the power *absorbed* by the element can be expressed by the product of the specified current and voltage variables. If the numerical value of the product is negative, then we say that the element is absorbing negative power, or that it is actually generating power and delivering it to some external element. For example, in Fig. 2.12 with $v = 5$ V and $i = -4$ A, the element may be described as either absorbing −20 W or generating 20 W.

If the current arrow is directed into the "+" marked terminal of an element, then p vi yields the *absorbed* power. A negative value indicates that power is actually being generated by the element.

If the current arrow is directed out of the "+" terminal of an element, then p vi yields the *supplied* power. A negative value in this case indicates that power is being absorbed.

Conventions are only required when there is more than one way to do something, and confusion may result when two different groups try to communicate. For example, it is rather arbitrary to always place "North" at the top of a map; compass needles don't point "up," anyway. Still, if we were talking to people who had secretly chosen the opposite convention of placing "South" at the top of their maps, imagine the confusion that could result! In the same fashion, there is a general convention that always draws the current arrows pointing into the positive voltage terminal, regardless of whether the element supplies or absorbs power. This convention is not incorrect but sometimes results in counterintuitive currents labeled on circuit schematics. The reason for this is that it simply seems more natural to refer to positive current flowing out of a voltage or current source that is supplying positive power to one or more circuit elements.

Compute the power absorbed by each part in Fig. 2.13.

3 A

+

2 V

–

(*a*)

–

–2 V

+

(*b*)

–3 A

–5 A

+

4 V

–

(*c*)

■ **FIGURE 2.13** (*a, b, c*) Three examples of two-terminal elements.

In Fig. 2.13*a*, we see that the reference current is defined consistent with the passive sign convention, which assumes that the element is absorbing power. With +3 A flowing into the positive reference termi nal, we compute

$$P = (2\text{ V})(3\text{ A}) = 6\text{ W}$$

of power absorbed by the element.

Figure 2.13*b* shows a slightly different picture. Now, we have a cur rent of −3 A flowing into the positive reference terminal. This gives us an absorbed power

$$P = (-2\text{ V})(-3\text{ A}) = 6\text{ W}$$

Thus, we see that the two cases are actually equivalent: A current of +3 A flowing into the top terminal is the same as a current of +3 A flowing out of the bottom terminal, or, equivalently, a current of −3 A flowing into the bottom terminal.

Referring to Fig. 2.13*c*, we again apply the passive sign convention rules and compute an absorbed power

$$P = (4\text{ V})(-5\text{ A}) = -20\text{ W}$$

Since we computed a negative *absorbed* power, this tells us that the element in Fig. 2.13*c* is actually *supplying* +20 W (i.e., it's a source of energy).

PRACTICE •

2.6 Determine the power being absorbed by the circuit element in Fig. 2.14*a*.

+

220 mV

4 A

–

–3.8 V

$8e^{-100t}$ V

(*a*)

–

+

– +

[redacted]1.75 A (*b*)

(*c*)

3.2 A

■ **FIGURE 2.14**

2.7 Determine the power being generated by the circuit element in Fig. 2.14*b*.

2.8 Determine the power being delivered to the circuit element in Fig. 2.14*c* at t = 5 ms.

Ans: 880 mW; 6.65 W; −15.53 W.

2.3 VOLTAGE AND CURRENT SOURCES

Using the concepts of current and voltage, it is now possible to be more spe cific in defining a ***circuit element.***

In so doing, it is important to differentiate between the physical device itself and the mathematical model which we will use to analyze its behavior in a circuit. The model is only an approximation.

Let us agree that we will use the expression *circuit element* to refer to the mathematical model. The choice of a particular model for any real device must be made on the basis of experimental data or experience; we will usually assume that this choice has already been made. For simplicity, we initially consider circuits with idealized components represented by simple models.

By definition, a simple circuit element is the mathematical model of a two-terminal electrical device, and it can be completely characterized by its voltage-current relationship; it cannot be subdivided into other two-terminal devices.

All the simple circuit elements that we will consider can be classified ac cording to the relationship of the current through the element to the voltage across the element. For example, if the voltage across the element is linearly proportional to the current through it, we will call the element a resistor. Other types of simple circuit elements have terminal voltages which are proportional to the *derivative* of the current with respect to time (an induc tor), or to the *integral* of the current with respect to time (a capacitor). There are also elements in which the voltage is completely independent of the cur rent, or the current is completely independent of the voltage; these are termed *independent sources*. Furthermore, we will need to define special kinds of sources for which either the source voltage or current depends upon a current or voltage elsewhere in the circuit; such sources are referred to as *dependent sources*. Dependent sources are used a great deal in electronics to model both dc and ac behavior of transistors, especially in amplifier circuits.

v_s + −
i

v_s + −
i

v_s + −

Independent Voltage Sources

The first element we will consider is the ***independent voltage source.*** The circuit symbol is shown in Fig. 2.15*a*; the subscript *s* merely identifies the voltage as a "source" voltage, and is common but not required. *An inde pendent voltage source is characterized by a terminal voltage which is completely independent of the current through it.*

(*a*) (*b*) (*c*)

■ **FIGURE 2.15** Circuit symbol of the independent voltage source.

If you've ever noticed the room lights dim when an air conditioner kicks on, it's because the sudden large current demand temporarily led to a voltage drop. After the motor starts moving, it takes less current to keep it in motion. At that point, the current demand is reduced, the voltage returns to its original value, and the wall outlet again provides a reasonable approximation of an ideal voltage source.

Thus, if we are given an independent voltage source and are notified that the terminal voltage is 12 V, then we always assume this voltage, regardless of the current flowing.

The independent voltage source is an *ideal* source and does not repre sent exactly any real physical device, because the ideal source could theo retically deliver an infinite amount of energy from its terminals. This ideal ized voltage source does, however, furnish a reasonable approximation to several practical voltage sources. An automobile storage battery, for exam ple, has a 12 V terminal voltage that remains essentially constant as long as the current through it does not exceed a few amperes. A small current may flow in either direction through the battery. If it is positive and flowing out of the positively marked terminal, then the battery is furnishing power to the headlights, for example; if the current is positive and flowing into the posi tive terminal, then the battery is charging by absorbing energy from the alternator.[3] An ordinary household electrical outlet also approximates an independent voltage source, providing a voltage $v_s = 115\sqrt{2}\cos 2\pi 60t$ V; this representation is valid for currents less than 20 A or so. A point worth repeating here is that the presence of the plus sign at the upper end of the symbol for the independent voltage source in Fig. 2.15*a* does not necessarily mean that the upper terminal is numerically positive with respect to the lower terminal. Instead, it means that the upper terminal is v_s volts positive with respect to the lower. If at some instant v_s happens to be negative, then the upper terminal is actually negative with respect to the lower at that instant.

(3) Or the battery of a friend's car, if you accidentally left your headlights on. . .

Consider a current arrow labeled "*i*" placed adjacent to the upper conduc tor of the source as in Fig. 2.15*b*. The current *i* is entering the terminal at which the positive sign is located, the passive sign convention is satisfied, and the source thus *absorbs* power $p = v_s i$. More often than not, a source is expected to deliver power to a network and not to absorb it. Consequently, we might choose to direct the arrow as in Fig. 2.15*c* so that $v_s i$ will represent the power *delivered* by the source. Technically, either arrow direction may be chosen; whenever possible, we will adopt the convention of Fig. 2.15*c* in this text for voltage and current

V_s + −
+ V −
v_s
+ −

(*a*) (*b*) (*c*)

sources, which are not usually considered passive devices. An independent voltage source with a constant terminal voltage is often termed an independent dc voltage source and can be represented by either of the symbols shown in Fig. 2.16*a* and *b*. Note in Fig. 2.16*b* that when the physical plate structure of the battery is suggested, the longer plate is placed at the positive terminal; the plus and minus signs then represent redundant notation, but they are usually included anyway. For the sake of complete ness, the symbol for an independent ac voltage source is shown in Fig. 2.16*c*.

Independent Current Sources

Another ideal source which we will need is the ***independent current source.*** Here, the current through the element is completely independent of the voltage across it. The symbol for an independent current source is shown in Fig. 2.17. If i_s is constant, we call the source an independent dc current source. An ac current source is often drawn with a tilde through the arrow, similar to the ac voltage source shown in Fig. 2.16*c*.

Like the independent voltage source, the independent current source is at best a reasonable approximation for a physical element. In theory it can deliver infinite power from its terminals because it produces the same finite current for any voltage across it, no matter how large that voltage may be. It is, however, a good approximation for many practical sources, particularly in electronic circuits. Although most students seem happy enough with an independent volt age source providing a fixed voltage but essentially any current, *it is a com mon mistake* to view an independent current source as having zero voltage across its terminals while providing a fixed current. In fact, we do not know a priori what the voltage across a current source will be—it depends entirely on the circuit to which it is connected.

Dependent Sources

The two types of ideal sources that we have discussed up to now are called *independent* sources because the value of the source quantity is not affected in any way by activities in the remainder of the circuit. This is in contrast with yet another kind of ideal source, the *dependent,* or *controlled,* source,

■ **FIGURE 2.16** (*a*) DC voltage source symbol; (*b*) battery symbol; (*c*) ac voltage source symbol.

Terms like dc voltage source and dc current source are commonly used. Literally, they mean "direct-current voltage source" and "direct-current current source," respectively. Although these terms may seem a little odd or even redundant, the terminology is so widely used there's no point in fighting it.

i_s

■ **FIGURE 2.17** Circuit symbol for the independent current source.

in which the source quantity is determined by a voltage or current existing at some other location in the system being analyzed. Sources such as these appear in the equivalent electrical models for many electronic devices, such as transistors, operational amplifiers, and integrated circuits. To distinguish between dependent and independent sources, we introduce the diamond symbols shown in Fig. 2.18. In Fig. 2.18*a* and *c*, *K* is a dimensionless scaling constant. In Fig.2.18*b*, *g* is a scaling factor with units of A/V; in Fig. 2.18*d*, *r* is a scaling factor with units of V/A. The controlling current i_x and the controlling voltage v_x must be defined in the circuit.

Ki_x

(*a*)

gv_x

(*b*)

Kv_x

+

−

ri_x

(*c*)

+

−

(*d*)

■ **FIGURE 2.18** The four different types of dependent sources: (*a*) current-controlled current source; (*b*) voltage-controlled current source; (*c*) voltage-controlled voltage source; (*d*) current controlled voltage source.

It does seem odd at first to have a current source whose value depends on a voltage, or a voltage source which is controlled by a current flowing through some other element. Even a voltage source depending on a remote voltage can appear strange. Such sources are invaluable for modeling com plex systems, however, making the analysis algebraically straightforward. Examples include the drain current of a field effect transistor as a function of the gate voltage, or the output voltage of an analog integrated circuit as a function of differential input voltage. When encountered during circuit analysis, we write down the entire controlling expression for the dependent source just as we would if it was a numerical value attached to an indepen dent source. This often results in the need for an additional equation to complete the analysis, unless the controlling voltage or current is already one of the specified unknowns in our system of equations.

In the circuit of Fig. 2.19*a*, if υ_2 is known to be 3 V, find υ_L.

We have been provided with a partially labeled circuit diagram and the additional information that $v_2 = 3$ V. This is probably worth adding to our diagram, as shown in Fig. 2.19*b*.

Next we step back and look at the information collected. In examin ing the circuit diagram, we notice that the desired voltage v_L is the same as the voltage across the dependent source. Thus, $v_L = 5v_2$

At this point, we would be done with the problem if only we knew v_2! Returning to our diagram, we see that we actually do know v_2—it was specified as 3 V. We therefore write

$v_2 = 3$

We now have two (simple) equations in two unknowns, and solve to find $v_L = 15$ V.

+ + + – υ_2 – – υ_L $5\upsilon_2$ – + (*a*)

+ + – + $\upsilon_2 = 3$ V – – υ_L (*b*) $5\upsilon_2$ –

■ **FIGURE 2.19** (*a*) An example circuit containing a voltage-controlled voltage source. (*b*) The additional information provided is included on the diagram.

An important lesson at this early stage of the game is that *the time it takes to completely label a circuit diagram is always a good invest ment.* As a final step, we should go back and check over our work to ensure that the result is correct.

PRACTICE •

2.9 Find the power *absorbed* by each element in the circuit in Fig. 2.20.

– + 7 A 8 V 8 V 0.25υ_x 5 A – υ_x 12 V + 20 V – + + 20 V + + – – – – 2 A 8 A

■ **FIGURE 2.20**

Ans: (left to right) −56 W; 16 W; −60 W; 160 W; −60 W.

Dependent and independent voltage and current sources are *active* ele ments; they are capable of delivering power to some external device. For the present we will think of a *passive* element as one which is capable only of receiving power. However, we will later see that several passive elements are able to store finite amounts of energy and then return that energy later to various external devices; since we still wish to call such elements passive, it will be necessary to improve upon our two definitions a little later.

Networks and Circuits

The interconnection of two or more simple circuit elements forms an elec trical ***network.*** If the network contains at least one closed path, it is also an electric ***circuit.*** Note: Every circuit is a network, but not all networks are circuits (see Fig. 2.21)!

+ − v_s

−

v_s

(*a*)

(*b*)

+

FIGURE 2.21 (*a*) A network that is not a circuit. (*b*) A network that is a circuit.

A network that contains at least one active element, such as an indepen dent voltage or current source, is an active network. A network that does not contain any active elements is a passive network.

We have now defined what we mean by the term ***circuit element,*** and we have presented the definitions of several specific circuit elements, the independent and dependent voltage and current sources. Throughout the remainder of the book we will define only five additional circuit elements: the resistor, inductor, capacitor, transformer, and the ideal operational ampli fier ("op amp," for short). These are all ideal elements. They are important because we may combine them into networks and circuits that represent real devices as accurately as we require. Thus, the transistor shown in Fig. 2.22*a* and *b* may be modeled by the voltage terminals designated v_{gs} and the single dependent current source of Fig. 2.22*c*. Note that the dependent current source produces a current that depends on a voltage elsewhere in the circuit. The parameter g_m, commonly referred to as the transconductance, is calculated using transistor-specific details as well as the operating point de termined by the circuit connected to the transistor. It is generally a small number, on the order of 10^{-2} to perhaps 10A/V. This model works pretty well as long as the frequency of any sinusoidal source is neither very large nor very small; the model can be modified to account for frequency-dependent

Metal (or polysilicon)

ondioxide (SiO_2 W)

S

G

n^+

Sou ce eg o r r i n
n^+
Channel
region L
Drain
region (b)
(body)
g s
D
B
p-type substrate
v_{gs}
+
−
(c)
d s
$g_m v_{gs}$
(a)

FIGURE 2.22 The Metal Oxide Semiconductor Field Effect Transistor (MOSFET). (*a*) An IRF540 N-channel power MOSFET in a TO-220 package, rated at 100 V and 22 A; (*b*) cross-sectional view of a basic MOSFET (*R. Jaeger, Microelectronic Circuit Design, McGraw-Hill, 1997*); (*c*) equivalent circuit model for use in ac circuit analysis.

effects by including additional ideal circuit elements such as resistors and capacitors.

Similar (but much smaller) transistors typically constitute only one small part of an integrated circuit that may be less than 2 mm × 2 mm square and 200 μm thick and yet contains several thousand transistors plus various resistors and capacitors. Thus, we may have a physical device that is about the size of one letter on this page but requires a model composed of ten thousand ideal simple circuit elements. We use this concept of "circuit modeling" in a number of electrical engineering topics covered in other courses, including electronics, energy conversion, and antennas.

2.4 OHM'S LAW

So far, we have been introduced to both dependent and independent voltage and current sources and were cautioned that they were *idealized* active elements that could only be approximated in a real circuit. We are now ready to meet another idealized element, the linear resistor. The resistor is the simplest passive element, and we begin our discussion by considering the work of an obscure German physicist, Georg Simon Ohm, who published a pamphlet in 1827 that described the results of one of the first efforts to measure currents and voltages, and to describe and relate them mathematically. One result was a statement of the fundamental relationship we now call ***Ohm's law,*** even though it has since been shown that this result was discovered 46 years earlier in England by Henry Cavendish, a brilliant semirecluse.

Ohm's law states that the voltage across conducting materials is directly proportional to the current flowing through the material, or

$$v = Ri \quad [4]$$

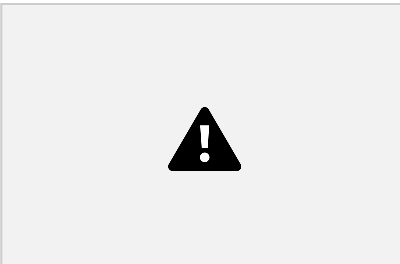
where the constant of proportionality R is called the ***resistance.*** The unit of resistance is the *ohm,* which is 1 V/A and customarily abbreviated by a capital omega, .

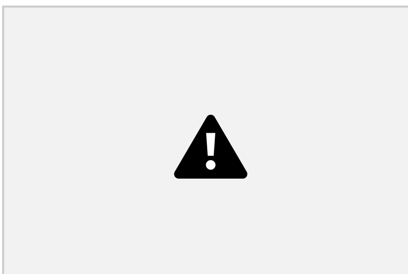
When this equation is plotted on i-versus-v axes, the graph is a straight line passing through the origin (Fig. 2.23). Equation [4] is a linear equation, and we will consider it as the definition of a *linear resistor*. Resistance is normally considered to be a positive quantity, although negative resistances may be simulated with special circuitry. Again, it must be emphasized that the linear resistor is an idealized circuit element; it is only a mathematical model of a real, physical device. "Resistors" may be easily purchased or manufactured, but it is soon found that the voltage-current ratios of these physical devices are reasonably constant only within certain ranges of current, voltage, or power, and depend also on temperature and other environmental factors. We usually refer to a linear resistor as simply a resistor; any resistor that is nonlinear will always be described as such. Nonlinear resistors should not necessarily be considered undesirable elements. Although it is true that their presence complicates an analysis, the performance of the device may depend on or be greatly improved by the nonlinearity. For example, fuses for overcurrent protection and Zener diodes for voltage regulation are very nonlinear in nature, a fact that is exploited when using them in circuit design.

Power Absorption

Figure 2.24 shows several different resistor packages, as well as the most common circuit symbol used for a resistor. In accordance with the voltage, current, and power conventions already adopted, the product of *v* and *i* gives the power absorbed by the resistor. That is, *v* and *i* are selected to satisfy the passive sign convention. The absorbed power appears physically

■ **FIGURE 2.23** Current-voltage relationship for an example 2 linear resistor. Note the slope of the line is 0.5 A/V, or 500 m [1].





■ **FIGURE 2.24** (*a*) Several common resistor packages. (*b*) A 560 power resistor rated at up to 50 W. (*c*) A 5% tolerance 10-teraohm (10,000,000,000,000) resistor manufactured by Ohmcraft. (*d*) Circuit symbol for the resistor, applicable to all of the devices in (*a*) through (*c*).

as heat and/or light and is always positive; a (positive) resistor is a passive element that cannot deliver power or store energy. Alternative expressions for the absorbed power are

$$p = vi = i^2R = v^2/R \quad [5]$$

One of the authors (who shall remain anonymous) had the unfortunate experience of inadvertently connecting a 100 , 2 W carbon resistor across a 110 V source. The ensuing flame, smoke, and fragmentation were rather disconcerting, demonstrating clearly that a practical resistor has definite limits to its ability to behave like the ideal linear model. In this case, the un fortunate resistor was called upon to absorb 121 W; since it was designed to handle only 2 W, its reaction was understandably violent.

The 560 resistor shown in Fig. 2.24*b* is connected to a circuit which causes a current of 42.4 mA to flow through it. Calculate the voltage across the resistor and the power it is dissipating.

The voltage across the resistor is given by Ohm's law:

$$v = Ri = (560)(0.0424) = 23.7 \text{ V}$$

The dissipated power can be calculated in several different ways. For instance,

$$p = vi = (23.7)(0.0424) = 1.005 \text{ W}$$

Alternatively,

$p = v^2/R = (23.7)^2/560 = 1.003$ W

or

$p = i^2R = (0.0424)^2(560) = 1.007$ W

We note several things.

First, we calculated the power in three different ways, and we seem to have obtained *three different answers*!

In reality, however, we rounded our voltage to three significant digits, which will impact the accuracy of any subsequent quantity we calculate with it. With this in mind, we see that the answers show rea sonable agreement (within 1%).

The other point worth noting is that the resistor is rated to 50 W— since we are only dissipating approximately 2% of this value, the resis tor is in no danger of overheating.

PRACTICE •

i

+ *v* −*R*

■ **FIGURE 2.25**

With reference to Fig. 2.25, compute the following:

2.10 R if $i = -2$ μA and $v = -44$ V.

2.11 The power absorbed by the resistor if $v = 1$ V and $R = 2$ kΩ.

2.12 The power absorbed by the resistor if $i = 3$ nA and $R = 4.7$ MΩ.

Ans: 22 M ; 500 μW; 42.3 pW.

~~PRACTICAL APPLICATION~~

~~PRACTICAL~~

~~APPLICATION~~ Wire Gauge

Technically speaking, any material (except for a super conductor) will provide resistance to current flow. As in all introductory circuits texts, however, we tacitly as sume that wires appearing in circuit diagrams have zero resistance. This implies that there is no potential differ ence between the ends of a wire, and hence no power absorbed or heat generated. Although usually not an unreasonable assumption, it does neglect practical con siderations when choosing the appropriate wire diameter for a specific application.

Resistance is determined by (1) the inherent resistiv ity of a material and (2) the device geometry. ***Resistivity,*** represented by the symbol ρ, is a measure of the ease with which electrons can travel through a certain mater ial. Since it is the ratio of the electric field (V/m) to the areal density of current flowing in the material (A/m^2), the general unit of ρ is an ·m, although metric pre fixes are often employed. Every material has a different inherent resistivity, which depends on temperature. Some examples are shown in Table 2.3; as can be seen, there is a small variation between different types of cop per (less than 1%) but a very large difference between different metals. In particular, although physically stronger than copper, steel wire is several times more resistive. In some technical discussions, it is more common to see the conductivity (symbolized by σ) of a material quoted, which is simply the reciprocal of the resistivity.

The resistance of a particular object is obtained by multiplying the resistivity by the length of the resistor and dividing by the cross-sectional area (A) as in Eq. [6]; these parameters are illustrated in Fig. 2.26.

$$R = \rho A \quad [6]$$

(cm)

Cross-sectional area = A cm^2 Resistivity = cm

Direction of current flow

■ **FIGURE 2.26** Definition of geometrical parameters used to compute the resistance of a wire. The resistivity of the material is assumed to be spatially uniform.

We determine the resistivity when we select the material from which to fabricate a wire and measure the temperature of the application environment. Since a finite amount of power is absorbed by the wire due to its resistance, current flow leads to the production of heat. Thicker wires have lower resistance and also dissipate heat more easily but are heavier, take up a larger volume, and are more expensive. Thus, we are motivated by practical considerations to choose the smallest wire that

TABLE 2.3 Common Electrical Wire Materials and Resistivities*

ASTM Specification**	Temper and Shape	Resistivity at 20°C (μ · cm)
B33	Copper, tinned soft, round	1.7654
B75	Copper, tube, soft, OF copper	1.7241
B188	Copper, hard bus tube, rectangular or square	1.7521
B189	Copper, lead-coated soft, round	1.7654
B230	Aluminum, hard, round	2.8625
B227	Copper-clad steel, hard, round, grade 40 HS	4.3971
B355	Copper, nickel-coated soft, round Class 10	1.9592
B415	Aluminum-clad steel, hard, round	8.4805

* C. B. Rawlins, "Conductor materials," *Standard Handbook for Electrical Engineering,* 13th ed., D. G. Fink and H. W. Beaty, eds. New York: McGraw-Hill, 1993, pp. 4-4 to 4-8.

** American Society of Testing and Materials.

(Continued on next page)

can safely do the job, rather than simply choosing the largest-diameter wire available in an effort to minimize resistive losses. The American Wire Gauge (AWG) is a standard system of specifying wire size. In selecting a wire gauge, smaller AWG corresponds to a larger wire diameter; an abbreviated table of common gauges is given in Table 2.4. Local fire and electrical safety codes typically dictate the required gauge for specific wiring applications, based on the maximum current expected as well as where the wires will be located.

TABLE 2.4 Some Common Wire Gauges and the Resistance of (Soft) Solid Copper Wire*

Conductor Size (AWG)	Cross-Sectional Area (mm^2)	Ohms per 1000 ft at 20°C
28	0.0804	65.3
24	0.205	25.7
22	0.324	16.2
18	0.823	6.39
14	2.08	2.52
12	3.31	1.59
6	13.3	0.3952
4	21.1	0.2485
2	33.6	0.1563

* C. B. Rawlins et al., *Standard Handbook for Electrical Engineering,* 13th ed., D. G. Fink and H. W. Beaty, eds. New York: McGraw-Hill, 1993, p. 4-47.

A dc power link is to be made between two islands separated by a distance of 24 miles. The operating voltage is 500 kV and the sys tem capacity is 600 MW. Calculate the maximum dc current flow, and estimate the resistivity of the cable, assuming a diameter of 2.5 cm and a solid (not stranded) wire.

Dividing the maximum power (600 MW, or 600×10^6 W) by the operating voltage (500 kV, or 500×10^3 V) yields a maximum current of

$$\frac{600 \times 10^6}{500 \times 10^3} = 1200 \text{ A}$$

The cable resistance is simply the ratio of the voltage to the current, or

$$R_{\text{cable}} = \frac{500 \times 10^3}{1200} = 417$$

2.54 cm

We know the length:

$$= (24 \text{ miles}) \; \frac{5280 \text{ ft}}{1 \text{ mile}} \; \frac{12 \text{ in}}{1 \text{ ft}} \; \frac{2.54 \text{ cm}}{1 \text{ in}} = 3{,}862{,}426 \text{ cm}$$

Given that most of our information appears to be valid to only two signif icant figures, we round this to 3.9×10^6 cm.

With the cable diameter specified as 2.5 cm, we know its cross-sectional area is 4.9 cm^2.

$$\text{Thus, } \rho_{\text{cable}} = R_{\text{cable}} \frac{A}{} = 417 \; \frac{4.9}{3.9 \times 10^6}$$

PRACTICE •

$= 520\ \mu \cdot$ cm

2.13 A 500 ft long 24 AWG soft copper wire is carrying a current of 100 mA. What is the voltage dropped across the wire?

Ans: 3.26 V.

Conductance

For a linear resistor the ratio of current to voltage is also a constant

$$\frac{i}{v} = \frac{1}{R} = G \quad [7]$$

where G is called the *conductance*. The SI unit of conductance is the siemens (S), 1 A/V. An older, unofficial unit for conductance is the mho, which was often abbreviated as and is still occasionally written as $^{-1}$. You will occasionally see it used on some circuit diagrams, as well as in cat alogs and texts. The same circuit symbol (Fig. 2.24*d*) is used to represent both resistance and conductance. The absorbed power is again necessarily positive and may be expressed in terms of the conductance by

$$p = vi = v^2 G = \frac{i^2}{G} \quad [8]$$

Thus a 2 resistor has a conductance of $\frac{1}{2}$ S, and if a current of 5 A is flowing through it, then a voltage of 10 V is present across the terminals and a power of 50 W is being absorbed.

All the expressions given so far in this section were written in terms of instantaneous current, voltage, and power, such as $v = iR$ and $p = vi$. We should recall that this is a shorthand notation for $v(t) = Ri(t)$ and $p(t) = v(t)i(t)$. The current through and voltage across a resistor must both vary with time in the same manner. Thus, if $R = 10$ and $v = 2 \sin 100t$ V, then $i = 0.2 \sin 100t$ A. Note that the power is given by $0.4 \sin^2 100t$ W, and a simple sketch will illustrate the different nature of its variation with time. Although the current and voltage are each negative during certain time intervals, the absorbed power is *never* negative!

Resistance may be used as the basis for defining two commonly used terms, *short circuit* and *open circuit*. We define a short circuit as a resistance of zero ohms; then, since $v = iR$, the voltage across a short circuit must be zero, although the current may have any value. In an analogous manner, Note that a current represented by *i* or *i*(*t*) can be constant (dc) or time-varying, but currents represented by the symbol *I* must be non-time-varying.

we define an open circuit as an infinite resistance. It follows from Ohm's law that the current must be zero, regardless of the voltage across the open circuit. Although real wires have a small resistance associated with them, we always assume them to have zero resistance unless otherwise specified. Thus, in all of our circuit schematics, wires are taken to be perfect short circuits.

SUMMARY AND REVIEW

In this chapter, we introduced the topic of units – specifically those relevant to electrical circuits—and their relationship to fundamental (SI) units. We also discussed current and current sources, voltage and voltage sources, and the fact that the product of voltage and current yields power (the rate of energy consumption or generation). Since power can be either positive or negative depending on the current direction and voltage polarity, the pas sive sign convention was described to ensure we always know if an element is *absorbing* or *supplying* energy to the rest of the circuit. Four additional sources were introduced, forming a general class known as dependent sources. They are often used to model complex systems and electrical com ponents, but the actual value of voltage or current supplied is typically unknown until the entire circuit is analyzed. We concluded the chapter with the resistor—by far the most common circuit element—whose voltage and current are linearly related (described by Ohm's law). Whereas the *resistiv ity* of a material is one of its fundamental properties (measured in · cm), *resistance* describes a device property (measured in) and hence depends not only on resistivity but on the device geometry (i.e., length and area) as well. We conclude with key points of this chapter to review, along with ap propriate examples.

❑ The system of units most commonly used in electrical engineering is the SI.

❑ The direction in which positive charges are moving is the direction of positive current flow; alternatively, positive current flow is in the direction opposite that of moving electrons.

❑ To define a current, both a value and a direction must be given. Currents are typically denoted by the uppercase letter "*I*" for constant (dc) values, and either *i(t)* or simply *i* otherwise.

❑ To define a voltage across an element, it is necessary to label the terminals with "+" and "−" signs as well as to provide a value (either an algebraic symbol or a numerical value).

❑ Any element is said to supply positive power if positive current flows out of the positive voltage terminal. Any element absorbs positive power if positive current flows into the positive voltage terminal. (Example 2.1)

❑ There are six sources: the independent voltage source, the independent current source, the current-controlled dependent current source, the voltage-controlled dependent current source, the voltage-controlled dependent voltage source, and the current-controlled dependent voltage source. (Example 2.2)

❑ Ohm's law states that the voltage across a linear resistor is directly proportional to the current flowing through it; i.e., $v = Ri$. (Example 2.3)

❑ The power dissipated by a resistor (which leads to the production of heat) is given by $p = vi = i^2R = v^2/R$. (Example 2.3)

❑ Wires are typically assumed to have zero resistance in circuit analysis.

When selecting a wire gauge for a specific application, however, local electrical and fire codes must be consulted. (Example 2.4)

READING FURTHER

A good book that discusses the properties and manufacture of resistors in considerable depth:

Felix Zandman, Paul-René Simon, and Joseph Szwarc, *Resistor Theory and Technology.* Raleigh, N.C.: SciTech Publishing, 2002.

A good all-purpose electrical engineering handbook:

Donald G. Fink and H. Wayne Beaty, *Standard Handbook for Electrical Engineers,* 13th ed., New York: McGraw-Hill, 1993.

In particular, pp. 1-1 to 1-51, 2-8 to 2-10, and 4-2 to 4-207 provide an in-depth treatment of topics related to those discussed in this chapter.

A detailed reference for the SI is available on the Web from the National Institute of Standards:

Ambler Thompson and Barry N. Taylor, *Guide for the Use of the International System of Units (SI),* NIST Special Publication 811, 2008 edition, www.nist.gov.

EXERCISES

2.1 Units and Scales

1. Convert the following to engineering notation:

   (*a*) 0.045 W (*b*) 2000 pJ

   (*c*) 0.1 ns (*d*) 39,212 as

   (*e*) 3 (*f*) 18,000 m

   (*g*) 2,500,000,000,000 bits (*h*) 10^{15} atoms/cm^3

2. Convert the following to engineering notation:

   (*a*) 1230 fs (*b*) 0.0001 decimeter

   (*c*) 1400 mK (*d*) 32 nm

   (*e*) 13,560 kHz (*f*) 2021 micromoles

   (*g*) 13 deciliters (*h*) 1 hectometer

3. Express the following in engineering units:

   (*a*) 1212 mV (*b*) 10^{11} pA

   (*c*) 1000 yoctoseconds (*d*) 33.9997 zeptoseconds

   (*e*) 13,100 attoseconds (*f*) 10^{-14} zettasecond

   (*g*) 10^{-5} second (*h*) 10^{-9} Gs

4. Expand the following distances in simple meters:

   (*a*) 1 Zm (*b*) 1 Em (*c*) 1 Pm

   (*d*) 1 Tm (*e*) 1 Gm (*f*) 1 Mm

5. Convert the following to SI units, taking care to employ proper engineering notation:

   (*a*) 212°F (*b*) 0°F (*c*) 0 K

   (*d*) 200 hp (*e*) 1 yard (*f*) 1 mile

6. Convert the following to SI units, taking care to employ proper engineering notation:

   (*a*) 100 C (*b*) 0 C (*c*) 4.2 K

   (*d*) 150 hp (*e*) 500 Btu (*f*) 100 J/s

7. A certain krypton fluoride laser generates 15 ns long pulses, each of which contains 550 mJ of energy. (*a*) Calculate the peak instantaneous output power of the laser. (*b*) If up to 100 pulses can be generated per second, calculate the maximum average power output of the laser.

8. When operated at a wavelength of 750 nm, a certain Ti:sapphire laser is capa ble of producing pulses as short as 50 fs, each with an energy content of 500 μJ. (*a*) Calculate the instantaneous output power of the laser. (*b*) If the laser is capable of a pulse repetition rate of 80 MHz, calculate the maximum average output power that can be achieved.
9. An electric vehicle is driven by a single motor rated at 40 hp. If the motor is run continuously for 3 h at maximum output, calculate the electrical energy consumed. Express your answer in SI units using engineering notation.
10. Under insolation conditions of 500 W/m^2 (direct sunlight), and 10% solar cell efficiency (defined as the ratio of electrical output power to incident solar power), calculate the area required for a photovoltaic (solar cell) array capable of running the vehicle in Exer. 9 at half power.
11. A certain metal oxide nanowire piezoelectricity generator is capable of producing 100 pW of usable electricity from the type of motion obtained from a person jogging at a moderate pace. (*a*) How many nanowire devices are required to operate a personal MP3 player which draws 1 W of power? (*b*) If the nanowires can be produced with a density of 5 devices per square micron directly onto a piece of fabric, what area is required, and would it be practical?
12. A particular electric utility charges customers different rates depending on their daily rate of energy consumption: \$0.05/kWh up to 20 kWh, and \$0.10/kWh for all energy usage above 20 kWh in any 24 hour period. (*a*) Calculate how many 100 W light bulbs can be run continuously for less than \$10 per week. (*b*) Calculate the daily energy cost if 2000 kW of power is used continuously.
13. The Tilting Windmill Electrical Cooperative LLC Inc. has instituted a differential pricing scheme aimed at encouraging customers to conserve electricity use during daylight hours, when local business demand is at its highest. If the price per kilowatthour is \$0.033 between the hours of 9 p.m. and 6 a.m., and \$0.057 for all other times, how much does it cost to run a 2.5 kW portable heater continuously for 30 days?
14. Assuming a global population of 9 billion people, each using approximately 100 W of power continuously throughout the day, calculate the total land area that would have to be set aside for photovoltaic power generation, assuming 800 W/m^2 of incident solar power and a conversion efficiency (sunlight to electricity) of 10%.

2.2 Charge, Current, Voltage, and Power

15. The total charge flowing out of one end of a small copper wire and into an unknown device is determined to follow the relationship $q(t) = 5e^{-t/2}$ C, where t is expressed in seconds. Calculate the current flowing into the device, taking note of the sign.
16. The current flowing into the collector lead of a certain bipolar junction transistor (BJT) is measured to be 1 nA. If no charge was transferred in or out of the collector lead prior to $t = 0$, and the current flows for 1 min, calculate the total charge which crosses into the collector.

-10^{13} C.

17. The total charge stored on a 1 cm diameter insulating plate is

(*a*) How many electrons are on the plate? (*b*) What is the areal density of electrons (number of electrons per square meter)? (*c*) If additional electrons are added to the plate from an external source at the rate of 10^6 electrons per second, what is the magnitude of the current flowing between the source and the plate?
18. A mysterious device found in a forgotten laboratory accumulates charge at a rate specified by the expression $q(t) = 9 - 10t$ C from the moment it is switched on. (*a*) Calculate the total charge contained in the device at $t = 0$. (*b*) Calculate the total charge contained at $t = 1$ s. (*c*) Determine the current flowing into the device at $t = 1$ s, 3 s, and 10 s.
19. A new type of device appears to accumulate charge according to the expression $q(t) = 10t^2 - 22t$ mC (t in s). (*a*) In the interval $0 \leq t < 5$ s, at what time does the current flowing into the device equal zero? (*b*) Sketch $q(t)$ and $i(t)$ over the interval $0 \leq t < 5$ s.

20. The current flowing through a tungsten-filament light bulb is determined to follow $i(t) = 114 \sin(100\pi t)$ A. (*a*) Over the interval defined by $t = 0$ and $t = 2$ s, how many times does the current equal zero amperes? (*b*) How much charge is transported through the light bulb in the first second?

21. The current waveform depicted in Fig. 2.27 is characterized by a period of 8 s. (*a*) What is the average value of the current over a single period? (*b*) If $q(0) = 0$, sketch $q(t)$, $0 < t < 20$ s.

$i(t)$

12
10
8
6
4
2

1 2 3 5 7 9 11 13 15 4 6 8 10 12 14 t (s)

■ **FIGURE 2.27** An example of a time-varying current.

22. The current waveform depicted in Fig. 2.28 is characterized by a period of 4 s. (*a*) What is the average value of the current over a single period? (*b*) Compute the average current over the interval $1 < t < 3$ s. (*c*) If $q(0) = 1$ C, sketch $q(t)$, $0 < t < 4$ s.

$i(t)$

4
3
2
1

1 2 3 4 5 6 78
t (s)

−1 – 2 – 3

■ **FIGURE 2.28** An example of a time-varying current.

23. A path around a certain electric circuit has discrete points labeled *A, B,C,* and *D*. To move an electron from points *A* to *C* requires 5 pJ. To move an electron from *B* to *C* requires 3 pJ. To move an electron from *A* to *D* requires 8 pJ. (*a*) What is the potential difference (in volts) between points *B* and *C*, assuming a "+" reference at *C*? (*b*) What is the potential difference (in volts) between points *B* and *D*, assuming a "+" reference at *D*? (*c*) What is the potential difference (in volts) between points *A* and *B* (again, in volts), assuming a "+" reference at *B*?

24. Two metallic terminals protrude from a device. The terminal on the left is the positive reference for a voltage called v_x (the other terminal is the negative reference). The terminal on the right is the positive reference for a voltage called v_y (the other terminal being the negative reference). If it takes 1 mJ of energy to push a single electron into the left terminal, determine the voltages v_x and v_y.

25. The convention for voltmeters is to use a black wire for the negative reference terminal and a red wire for the positive reference terminal. (*a*) Explain why two wires are required to measure a voltage. (*b*) If it is dark and the wires into the voltmeter are swapped by accident, what will happen during the next measurement?

26. Determine the power absorbed by each of the elements in Fig. 2.29.

1 pA

10 mA

+

–

+ – 6 V

2 A

2 A 10 V

+ – 1 V

(*a*) (*b*) (*c*) ■ **FIGURE 2.29** Elements for Exer. 26.

27. Determine the power absorbed by each of the elements in Fig. 2.30.

$8e^{-t}$ mA

+ –

1 A

$10^{-3}\,i_1$

–

+

$-16e^{-t}$ V

2 V 2 V (t = 500 ms)

+

–

(i_1 = 100 mA)

(*a*)

(*b*) (*c*)

■ **FIGURE 2.30** Elements for Exer. 27.

28. A constant current of 1 ampere is measured flowing into the positive reference terminal of a pair of leads whose voltage we'll call v_p. Calculate the absorbed power at t = 1 s if $v_p(t)$ equals (*a*) +1 V; (*b*) −1 V; (*c*) 2 + 5 cos(5*t*) V; (*d*) $4e^{-2t}$ V, (*e*) Explain the significance of a negative value for absorbed power.

2 A

5 A +

29. Determine the power supplied by the leftmost element in the circuit of Fig. 2.31.

– +

2 V –

–

8 V

–3 A

–4 A 10 V –

– + 10 V

+

+

■ **FIGURE 2.31**

30. The current-voltage characteristic of a silicon solar cell exposed to direct sunlight at noon in Florida during midsummer is given in Fig. 2.32. It is obtained by placing different-sized resistors across the two terminals of the device and measuring the resulting currents and voltages.

(*a*) What is the value of the short-circuit current?

(*b*) What is the value of the voltage at open circuit?

(*c*) Estimate the maximum power that can be obtained from the device.

Current (A)

3.0

2.5

2.0

1.5

1.0

0.5

0.125 0.250 0.375 0.500 Voltage (V)

■ **FIGURE 2.32**

2.3 Voltage and Current Sources

31. Some of the ideal sources in the circuit of Fig. 2.31 are supplying positive power, and others are absorbing positive power. Determine which are which, and show that the algebraic sum of the power absorbed by each element (taking care to preserve signs) is equal to zero.

32. By careful measurements it is determined that a benchtop argon ion laser is consuming (absorbing) 1.5 kW of electric power from the wall outlet, but only producing 5 W of optical power. Where is the remaining power going? Doesn't conservation of energy require the two quantities to be equal?

33. Refer to the circuit represented in Fig. 2.33, while noting that the same current flows through each element. The voltage-controlled dependent source provides a current which is 5 times as large as the voltage V_x. (a) For $V_R = 10$ V and $V_x = 2$ V, determine the power absorbed by each element. (*b*) Is element *A* likely a passive or active source? Explain.

+ V_R −

A

−

V_x $5V_x$ + −

8 V

+

■ **FIGURE 2.33**

34. Refer to the circuit represented in Fig. 2.33, while noting that the same current flows through each element. The voltage-controlled dependent source provides a current which is 5 times as large as the voltage V_x. (*a*) For $V_R = 100$ V and $V_x = 92$ V, determine the power supplied by each element. (*b*) Verify that the algebraic sum of the supplied powers is equal to zero.

35. The circuit depicted in Fig. 2.34 contains a dependent current source; the magnitude and direction of the current it supplies are directly determined by the voltage labeled v_1. Note that therefore $i_2 = -3v_1$. Determine the voltage v_1 if $v_2 = 33i_2$ and $i_2 = 100$ mA.

+ i_2 v_2 −

+ − v_S v_1 − $3v_1$

+

■ **FIGURE 2.34**

36. To protect an expensive circuit component from being delivered too much power, you decide to incorporate a fast-blowing fuse into the design. Knowing that the circuit component is connected to 12 V, its minimum power consumption is 12 W, and the maximum power it can safely dissipate is 100 W, which of the three available fuse ratings should you select: 1 A, 4 A, or 10 A? Explain your answer.

37. The dependent source in the circuit of Fig. 2.35 provides a voltage whose value depends on the current i_x. What value of i_x is required for the dependent source to be supplying 1 W?

i_x

+

$-2i_x$ + − v_2 −

■ **FIGURE 2.35**

2.4 Ohm's Law

38. Determine the magnitude of the current flowing through a 4.7 k resistor if the voltage across it is (*a*) 1 mV; (*b*) 10 V; (*c*) $4e^{-t}$ V; (*d*) 100 cos(5*t*) V; (*e*) −7 V. 39. Real resistors can only be manufactured to a specific tolerance, so that in effect the value of the resistance is uncertain. For example, a 1   resistor specified as 5% tolerance could in practice be found to have a value anywhere in the range of 0.95 to 1.05 . Calculate the voltage across a 2.2 k 10% tolerance resistor if the current flowing through the element is (*a*) 1 mA; (*b*) 4 sin 44*t* mA.

40. (*a*) Sketch the current-voltage relationship (current on the *y*-axis) of a 2 k resistor over the voltage range of $-10 \text{ V} \leq V_{resistor} \leq +10$ V. Be sure to label both axes appropriately. (*b*) What is the numerical value of the slope (express your answer in siemens)?

41. Sketch the voltage across a 33   resistor over the range $0 < t < 2\pi$ s, if the current is given by 2.8 cos(*t*) A. Assume both the current and voltage are defined according to the passive sign convention.

42. Figure 2.36 depicts the current-voltage characteristic of three different resistive elements. Determine the resistance of each, assuming the voltage and current are defined in accordance with the passive sign convention.

0.05
–0.01
–0.02
–0.03
Current (mA)
0.04
0.03
0.02
0.01
0.00
–0.04
–0.05
–5 –4 –3 –2 –1
0 1 2 3 4 5 Voltage (V)

0.05
–0.01
–0.02
–0.03
Current (mA)
0.04
0.03
0.02
0.01
0.00
–0.04
–0.05
–5 –4 –3 –2 –1
0 1 2 3 4 5 Voltage (V)

(*a*) (*b*)

0.05
0.04
–0.04
–0.05
Current (mA)
–5 –4 –3 –2 –1 0 1 2 3 4 5
Voltage (V)

■ **FIGURE 2.36**

0.03
0.02
0.01
0.00
–0.01
–0.02
–0.03

(*c*)

43. Determine the conductance (in siemens) of the following: (*a*) 0 ; (*b*) 100 M ; (*c*) 200 m .

44. Determine the magnitude of the current flowing through a 10 mS conductance if the voltage across it is (*a*) 2 mV; (*b*) −1 V; (*c*) $100e^{-2t}$ V; (*d*) 5 sin(5*t*) V; (*e*) 0 V.

45. A 1% tolerance 1 k resistor may in reality have a value anywhere in the range of 990 to 1010 . Assuming a voltage of 9 V is applied across it, determine (*a*) the corresponding range of current and (*b*) the corresponding

range of absorbed power. (*c*) If the resistor is replaced with a 10% tolerance 1 k resistor, repeat parts (*a*) and (*b*).

46. The following experimental data is acquired for an unmarked resistor, using a variable-voltage power supply and a current meter. The current meter readout is somewhat unstable, unfortunately, which introduces error into the measurement.

Voltage (V)	Current (mA)
−2.0	−0.89
−1.2	−0.47
0.0	0.01
1.0	0.44
1.5	0.70

(*a*) Plot the measured current-versus-voltage characteristic.

(*b*) Using a best-fit line, estimate the value of the resistance.

+
R_1
R_2 $^{+}_{-}$ V_S V_{R2} −

■ **FIGURE 2.37**

47. Utilize the fact that in the circuit of Fig. 2.37, the total power supplied by the voltage source must equal the total power absorbed by the two resistors to show that

$$V_{R2} = V_S \frac{R_2}{R_1 + R_2}$$

You may assume the same current flows through each element (a requirement of charge conservation).

48. For each of the circuits in Fig. 2.38, find the current I and compute the power absorbed by the resistor.

10 k
− I +
$^{+}_{-}$ I 5 V
5 V
10 k
10 k
− + I
$^{+}_{-}$ I −5 V
10 k
–5 V

■ **FIGURE 2.38**

49. Sketch the power absorbed by a 100 resistor as a function of voltage over the range −2 V ≤ $V_{resistor}$ ≤ +2 V.

Chapter-Integrating Exercises

50. So-called "*n*-type" silicon has a resistivity given by $\rho = (-q\, N_D \mu_n)^{-1}$, where N_D is the volume density of phosphorus atoms (atoms/cm^3), μ_n is the electron mobility (cm^2/V ·s), and $q = -1.602 \times 10^{-19}$ C is the charge of each electron. Conveniently, a relationship exists between mobility and N_D, as shown in Fig. 2.39. Assume an 8 inch diameter silicon wafer (disk) having a thickness of 300 μm. Design a 10 resistor by specifying a phosphorus concentration in the range of 2×10^{15} cm^{-3} ≤ N_D ≤ 2×10^{17} cm^{-3}, along with a suitable geometry (the wafer may be cut, but not thinned).

10^4

μ_n (cm^2/Vs)

10^3

10^{14} 10^{15} 10^{16} 10^{17} 10^{18} 10^{19} 10^2 N_D (atoms/cm^3)

■ **FIGURE 2.39**

51. Figure 2.39 depicts the relationship between electron mobility μ_n and dopant density N_D for n-type silicon. With the knowledge that resistivity in this material is given by $\rho = N_D\mu_n/q$, plot resistivity as a function of dopant density over the range $10^{14}\ \text{cm}^{-3} \leq N_D \leq 10^{19}\ \text{cm}^{-3}$.

52. Referring to the data of Table 2.4, design a resistor whose value can be varied mechanically in the range of 100 to 500 (assume operation at 20°C).
53. A 250 ft long span separates a dc power supply from a lamp which draws 25 A of current. If 14 AWG wire is used (note that two wires are needed for a total of 500 ft), calculate the amount of power wasted in the wire.
54. The resistance values in Table 2.4 are calibrated for operation at 20°C. They may be corrected for operation at other temperatures using the relationship[4]

$$\frac{R_2}{R_1} = \frac{234.5 + T_2}{234.5 + T_1}$$

where T_1 = reference temperature (20°C in present case)
T_2 = desired operating temperature
R_1 = resistance at T_1
R_2 = resistance at T_2

A piece of equipment relies on an external wire made of 28 AWG soft copper, which has a resistance of 50.0 at 20°C. Unfortunately, the operating environment has changed, and it is now 110.5°F. (*a*) Calculate the length of the original wire. (*b*) Determine by how much the wire should be shortened so that it is once again 50.0 .

55. Your favorite meter contains a precision (1% tolerance) 10 resistor. Unfortunately, the last person who borrowed this meter somehow blew the resistor, and it needs to be replaced. Design a suitable replacement, assuming at least 1000 ft of each of the wire gauges listed in Table 2.4 is readily available to you.
56. At a new installation, you specified that all wiring should conform to the ASTM B33 specification (see Table 2.3). Unfortunately the subcontractor misread your instructions and installed B415 wiring instead (but the same gauge). Assuming the operating voltage is unchanged, (*a*) by how much will the current be reduced, and (*b*) how much additional power will be wasted in the lines? (*Express both answers in terms of percentage.*)
57. If 1 mA of current is forced through a 1 mm diameter, 2.3 meter long piece of hard, round, aluminum-clad steel (B415) wire, how much power is wasted as a result of resistive losses? If instead wire of the same dimensions but conforming to B75 specifications is used, by how much will the power wasted due to resistive losses be reduced?
58. The network shown in Fig. 2.40 can be used to accurately model the behavior of a bipolar junction transistor provided that it is operating in the forward active mode. The parameter β is known as the current gain. If for this device

I_C

Collector

$\text{b}I_B$

0.7 V I_B

–
+
Base

Emitter

■ **FIGURE 2.40** DC model for a bipolar junction transistor operating in forward active mode.

(4) D. G. Fink and H. W. Beaty, *Standard Handbook for Electrical Engineers,* 13th ed. New York: McGraw-Hill, 1993, p. 2–9.

$\beta = 100$, and I_B is determined to be 100 μA, calculate (*a*) I_C, the current flowing into the collector terminal; and (*b*) the power dissipated by the base emitter region.

59. A 100 W tungsten filament light bulb functions by taking advantage of resistive losses in the filament, absorbing 100 joules each second of energy from the wall socket. How much *optical* energy per second do you expect it to produce, and does this violate the principle of energy conservation?

60. Batteries come in a wide variety of types and sizes. Two of the most common are called "AA" and "AAA." A single battery of either type is rated to produce a terminal voltage of 1.5 V when fully charged. So what are the differences between the two, other than size? (*Hint:* Think about energy.)

Voltag e and Curren t Laws

INTRODUCTION

In Chap. 2 we were introduced to independent voltage and current sources, dependent sources, and resistors. We discovered that *dependent* sources come in four varieties, and are controlled by a voltage or current which exists elsewhere. Once we know the voltage across a resistor, we know its current (and vice versa); this is not the case for sources, however. In general, circuits must be analyzed to determine a complete set of voltages and currents. This turns out to be reasonably straightforward, and only two simple laws are needed in addition to Ohm's law. These new laws are Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL), and they are simply restatements of charge and energy conservation, respectively. They apply to any circuit we will ever encounter, although in later chapters we will learn more efficient techniques for specific types of situations.

3.1 NODES, PATHS, LOOPS, AND BRANCHES

We now focus our attention on the current-voltage relationships in simple networks of two or more circuit elements. The elements will be connected by wires (sometimes referred to as "leads"), which have zero resistance. Since the network then appears as a number of sim ple elements and a set of connecting leads, it is called a ***lumped parameter network.*** A more difficult analysis problem arises when we are faced with a ***distributed-parameter network,*** which contains an essentially infinite number of vanishingly small elements. We will concentrate on lumped-parameter networks in this text.

Connections

Kirchhoff's Current Law (KCL)

Kirchhoff's Voltage Law (KVL)

Analysis of Basic Series and Parallel Circuits

Combination of Series and Parallel Sources

Reduction of Series and Parallel Resistor Combinations

New Circuit Terms: *Node, Path, Loop,* and *Branch*

Voltage and Current Division Ground

A point at which two or more elements have a common connection is called a ***node.*** For example, Fig. 3.1*a* shows a circuit containing three nodes. Sometimes networks are drawn so as to trap an unwary student into believing that there are more nodes present than is actually the case. This occurs when a node, such as node 1 in Fig. 3.1*a*, is shown as two separate junctions connected by a (zero-resistance) conductor, as in Fig. 3.1*b*. How ever, all that has been done is to spread the common point out into a common zero-resistance line.

In circuits assembled in the real world, the wires will always have finite resistance. However, this resistance is typically so small compared to other resistances in the circuit that we can neglect it without introducing significant error. In our idealized circuits, we will therefore refer to "zero resistance" wires from now on.

1

3

2

(*a*)

1

3

2

(*b*)

FIGURE 3.1 (*a*) A circuit containing three nodes and five branches. (*b*) Node 1 is redrawn to look like two nodes; it is still one node.

Thus, we must necessarily consider all of the perfectly conducting leads or portions of leads attached to the node as part of the node. Note also that every element has a node at each of its ends.

Suppose that we start at one node in a network and move through a sim ple element to the node at the other end. We then continue from that node through a different element to the next node, and continue this movement until we have gone through as many elements as we wish. If no node was encountered more than once, then the set of nodes and elements that we have passed through is defined as a ***path.*** If the node at which we started is the same as the node on which we ended, then the path is, by definition, a closed path or a ***loop.***

For example, in Fig. 3.1*a*, if we move from node 2 through the current source to node 1, and then through the upper right resistor to node 3, we have established a path; since we have not continued on to node 2 again, we have not made a loop. If we proceeded from node 2 through the current source to node 1, down through the left resistor to node 2, and then up through the central resistor to node 1 again, we do not have a path, since a node (actually two nodes) was encountered more than once; we also do not have a loop, because a loop must be a path.

Another term whose use will prove convenient is ***branch.*** We define a branch as a single path in a network, composed of one simple element and the node at each end of that element. Thus, a path is a particular collection of branches. The circuit shown in Fig. 3.1*a* and *b* contains five branches.

3.2 KIRCHHOFF'S CURRENT LAW

We are now ready to consider the first of the two laws named for Gustav Robert Kirchhoff (two *h*'s and two *f*'s), a German university professor who was born about the time Ohm was doing his experimental work. This axiomatic law is called Kirchhoff's current law (abbreviated KCL), and it simply states that

The algebraic sum of the currents entering any node is zero.

This law represents a mathematical statement of the fact that charge cannot accumulate at a node. *A node is not a circuit element,* and it certainly cannot store, destroy, or generate charge. Hence, the currents must sum to zero. A hydraulic analogy is sometimes useful here: for example, consider three water pipes joined in the shape of a Y. We define three "currents" as flowing *into* each of the three pipes. If we insist that water is always flow ing, then obviously we cannot have three positive water currents, or the pipes would burst. This is a result of our defining currents independent of

the direction that water is actually flowing. Therefore, the value of either one or two of the currents as defined must be negative.

Consider the node shown in Fig. 3.2. The algebraic sum of the four cur rents entering the node must be zero:

i_B i_A

i_D

i_C

$i_A + i_B + (-i_C) + (-i_D) = 0$

However, the law could be equally well applied to the algebraic sum of the currents *leaving* the node:

$(-i_A) + (-i_B) + i_C + i_D = 0$

We might also wish to equate the sum of the currents having reference arrows directed into the node to the sum of those directed out of the node:

$i_A + i_B = i_C + i_D$

which simply states that the sum of the currents going in must equal the sum of the currents going out.

current source. We are told the current out of the 10 V source is 3 A.

■ **FIGURE 3.2** Example node to illustrate the applica tion of Kirchhoff's current law.

For the circuit in Fig. 3.3*a*, compute the current through resistor R_3 if it is known that the voltage source supplies a current of 3 A.

R_1

2 A *i*

+ − R_2 R_3

10 V 5 A

(*a*)

Identify the goal of the problem.

The current through resistor R_3, labeled as *i* on the circuit diagram.

Collect the known information.

The node at the top of R_3 is connected to four branches. Two of these currents are clearly labeled: 2 A flows out of the node into R_2, and 5 A flows into the node from the

i_{R1}

R_1

2 A *i*

+ − R_2 R_3

10 V 5 A (*b*)

Devise a plan.

If we label the current through R_1 (Fig. 3.3*b*), we may write a KCL equation at the top node of resistors R_2 and R_3.

Summing the currents flowing into the node:

Construct an appropriate set of equations.

$i_{R1} - 2 - i + 5 = 0$

5 A

The currents flowing into this node are shown in the expanded dia gram of Fig. 3.3*c* for clarity.

i_{R1} ($i_{R1} - 2$ A)

Determine if additional information is required. We have one equation but two unknowns, which means we need to obtain an additional equation. At this point, the fact that we know the

R_1 5 A 2 A i R_2 R_3

10 V source is supplying 3 A comes in handy: KCL shows us that this is also the current i_{R_1}.

Attempt a solution.
Substituting, we find that $i = 3 - 2 + 5 = 6$ A.

Verify the solution. Is it reasonable or expected? It is always worth the effort to recheck our work. Also, we can attempt to evaluate whether at least the magnitude of the solution is

(*c*)

■ **FIGURE 3.3** (*a*) Simple circuit for which the current through resistor R_3 is desired. (*b*) The current through resistor R_1 is labeled so that a KCL equation can be written. (*c*) The currents into the top node of R_3 are redrawn for clarity.

(Continued on next page)

reasonable. In this case, we have two sources—one supplies 5 A, and the other supplies 3 A. There are no other sources, independent or dependent. Thus, we would not expect to find any current in the circuit in excess of 8 A.

PRACTICE •

3.1 Count the number of branches and nodes in the circuit in Fig. 3.4. If $i_x = 3$ A and the 18 V source delivers 8 A of current, what is the value of R_A? (*Hint:* You need Ohm's law as well as KCL.)

13 A

5

+

+ − v R_A x

18 V 6 −

i_x

■ **FIGURE 3.4**

Ans: 5 branches, 3 nodes, 1 .

A compact expression for Kirchhoff's current law is

$$\sum_{n=1}^{N} i_n = 0 \quad [1]$$

which is just a shorthand statement for

$$i_1 + i_2 + i_3 + \cdots + i_N = 0 \quad [2]$$

When Eq. [1] or Eq. [2] is used, it is understood that the N current arrows are either all directed toward the node in question, or are all directed away from it.

3.3 • KIRCHHOFF'S VOLTAGE LAW

Current is related to the charge flowing *through* a circuit element, whereas voltage is a measure of potential energy difference *across* the element. There is a single unique value for any voltage in circuit theory. Thus, the en ergy required to move a unit charge from point A to point B in a circuit must have a value independent of the path chosen to get from A to B (there is often more than one such path). We may assert this fact through Kirchhoff's voltage law (abbreviated **KVL**):

A C 2

v + 2 −
+
−

The algebraic sum of the voltages around any closed path is zero.

v_1 −
1
3

B

v_3
+

■ **FIGURE 3.5** The potential difference between points *A* and *B* is independent of the path selected.

In Fig. 3.5, if we carry a charge of 1 C from *A* to *B* through element 1, the reference polarity signs for v_1 show that we do v_1 joules of work.[1] Now

(1) Note that we chose a 1 C charge for the sake of numerical convenience: therefore, we did $(1\ \text{C})(v_1\ \text{J/C}) = v_1$ joules of work.

if, instead, we choose to proceed from *A* to *B* via node *C*, then we expend $(v_2 - v_3)$ joules of energy. The work done, however, is independent of the path in a circuit, and so any route must lead to the same value for the volt age. In other words,

$$v_1 = v_2 - v_3 \quad [3]$$

It follows that if we trace out a closed path, the algebraic sum of the volt ages across the individual elements around it must be zero. Thus, we may write

$$v_1 + v_2 + v_3 + \cdots + v_N = 0$$

or, more compactly,

$$^{N}_{n=1} \qquad v_n = 0 \quad [4]$$

We can apply KVL to a circuit in several different ways. One method that leads to fewer equation-writing errors than others consists of moving mentally around the closed path in a clockwise direction and writing down directly the voltage of each element whose (+) terminal is entered, and writing down the negative of every voltage first met at the (−) sign. Apply ing this to the single loop of Fig. 3.5, we have

$$-v_1 + v_2 - v_3 = 0$$

which agrees with our previous result, Eq. [3].

In the circuit of Fig. 3.6, find v_x and i_x.

We know the voltage across two of the three elements in the circuit. Thus, KVL can be applied immediately to obtain v_x.

Beginning with the bottom node of the 5 V source, we apply KVL clockwise around the loop:

$$-5 - 7 + v_x = 0$$

7 V
− +
+
+
5 V
i_x
−

so $v_x = 12$ V.

KCL applies to this circuit, but only tells us that the same current

100 v_x −

(i_x) flows through all three elements. We now know the voltage across the 100 resistor, however.

Invoking Ohm's law,

$i_x = v_x$

100 = 12

100A = 120 mA

PRACTICE •

1 V

+ −

+

■ **FIGURE 3.6** A simple circuit with two voltage sources and a single resistor.

3.2 Determine i_x and v_x in the circuit of Fig. 3.7. Ans: $v_x = -4$ V; $i_x = -400$ mA.

3 V

− +

i_x

10 v_x −

■ **FIGURE 3.7**

In the circuit of Fig. 3.8 there are eight circuit elements. Find v_{R2} (the voltage across R_2) and the voltage labeled v_x.

The best approach for finding v_{R2} is to look for a loop to which we can apply KVL. There are several options, but the leftmost loop offers a straightforward route, as two of the voltages are clearly specified. Thus, we find v_{R2} by writing a KVL equation around the loop on the left, starting at point *c*:

$$4 - 36 + v_{R2} = 0$$

which leads to $v_{R2} = 32$ V.

a 36 V

−

− +

+ 14 V −

v_x −

R_1

− +

−

4 V

R_2

v_{R2}

v_{s1}

+ − 12 V

v_2 +−

+

−

+

+

v_{R1}

Points *b* and *c*, as well as the wire between them, are all part of the same node.

c b

■ **FIGURE 3.8** A circuit with eight elements for which we desire v_{R2} and v_x.

To find v_x, we might think of this as the (algebraic) sum of the volt ages across the three elements on the right. However, since we do not have values for these quantities, such an approach would not lead to a numerical answer. Instead, we apply KVL beginning at point *c*, moving up and across the top to *a*, through v_x to *b*, and through the conducting lead to the starting point:

$$+4 - 36 + 12 + 14 + v_x = 0$$

so that

$$v_x = 6 \text{ V}$$

An alternative approach: Knowing v_{R2}, we might have taken the shortcut through R_2:

$$-32 + 12 + 14 + v_x = 0$$

yielding $v_x = 6$ V once again.

PRACTICE •

3.3 For the circuit of Fig. 3.9, determine (*a*) v_{R2} and (*b*) v_2, if $v_{R1} = 1$ V.

+ – 7 V
a
– +
+
–
– +
12 V – 9 V +
+
3 V
v_2 + –
+
–8 V +
v_{R2} –
v_x –
R_2
R_1
v_{R1} –

c b ■ **FIGURE 3.9**

Ans: (*a*) 4 V; (*b*) −8 V.

As we have just seen, the key to correctly analyzing a circuit is to first me thodically label all voltages and currents on the diagram. This way, carefully written KCL or KVL equations will yield correct relationships, and Ohm's law can be applied as necessary if more unknowns than equations are ob tained initially. We illustrate these principles with a more detailed example.

Determine v_x in the circuit of Fig. 3.10*a*.

5 A
4
+
8
+
– v_x
60 V 10 2 i_x –

(*a*)

5 A
i_4
+
+
8 4
v_4 + v_8 – + –
+
– v_x
60 V 10 2 i_x v_{10}
– i_{10} i_2 (*b*) –

■ **FIGURE 3.10** (*a*) A circuit for which v_x is to be determined using KVL. (*b*) Circuit with voltages and currents labeled.

We begin by labeling voltages and currents on the rest of the elements in the circuit (Fig. 3.10*b*). Note that v_x appears across the 2 resistor and the source i_x as well.

If we can obtain the current through the 2 resistor, Ohm's law will yield v_x. Writing the appropriate KCL equation, we see that $i_2 = i_4 + i_x$

Unfortunately, we do not have values for any of these three quanti ties. Our solution has (temporarily) stalled.

Since we were given the current flowing from the 60 V source, per haps we should consider starting from that side of the circuit. Instead of finding v_x using i_2, it might be possible to find v_x directly using KVL. We can write the following KVL equations:

$$-60 + v_8 + v_{10} = 0$$

and

$$-v_{10} + v_4 + v_x = 0 \quad [5]$$

This is progress: we now have two equations in four unknowns, an improvement over one equation in which *all* terms were unknown. In fact, we know that $v_8 = 40$ V through Ohm's law, as we were told that 5 A flows through the 8 resistor. Thus, $v_{10} = 0 + 60 - 40 = 20$ V,

(*Continued on next page*)

so Eq. [5] reduces to

$$v_x = 20 - v_4$$

If we can determine v_4, the problem is solved.

The best route to finding a numerical value for the voltage v_4 in this case is to employ Ohm's law, which requires a value for i_4. From KCL, we see that

$$i_4 = 5 - i_{10} = 5 - \frac{v_{10}}{10} = 5 - \frac{20}{10} = 3$$

so that $v_4 = (4)(3) = 12$ V and hence $v_x = 20 - 12 = 8$ V.

PRACTICE •

3.4 Determine v_x in the circuit of Fig. 3.11.

2 A
2
+
8

R_1
+
R_2
–
+
– v_x
30 V 10 2 i_x
–

+
–

v_{s1}
(*a*)

■ **FIGURE 3.11**

Ans: v_x = 12.8 V.

3.4 THE SINGLE-LOOP CIRCUIT

We have seen that repeated use of KCL and KVL in conjunction with Ohm's law can be applied to nontrivial circuits containing several loops and a num ber of different elements. Before proceeding further, this is a good time to focus on the concept of series (and, in the next section, parallel) circuits, as they form the basis of any network we will encounter in the future.

All of the elements in a circuit that carry the same current are said to be

+ −

connected in ***series.*** As an example, consider the circuit of Fig. 3.10. The

v_{s2}

60 V source is in series with the 8 resistor; they carry the same 5 A cur rent. However, the 8 resistor is not in series with the 4 resistor; they carry different currents. Note that elements may carry equal currents and not be in series; two 100 W light bulbs in neighboring houses may very well

v

+ − $R1$

(*b*)

i

+ −

carry equal currents, but they certainly do not carry the same current and are *not* connected in series.

Figure 3.12*a* shows a simple circuit consisting of two batteries and two resistors. Each terminal, connecting lead, and solder glob is assumed to

i i

v_{s1}

R_1

+

R_2

v_{s2}

have zero resistance; together they constitute an individual node of the circuit

+

diagram in Fig. 3.12*b*. Both batteries are modeled by ideal voltage sources; v_{R2}

−

−

i

(*c*)

■ **FIGURE 3.12** (*a*) A single-loop circuit with four elements. (*b*) The circuit model with source voltages and resistance values given. (*c*) Current and voltage reference signs have been added to the circuit.

any internal resistances they may have are assumed to be small enough to neglect. The two resistors are assumed to be ideal (linear) resistors. We seek the current *through* each element, the voltage *across* each ele ment, and the power *absorbed* by each element. Our first step in the analy sis is the assumption of reference directions for the unknown currents. Arbitrarily, let us select a clockwise current i which flows out of the upper terminal of the voltage source on the left. This choice is indicated by an ar row labeled i at that point in the circuit, as shown in Fig. 3.12*c*. A trivial

application of Kirchhoff's current law assures us that this same current must also flow through every other element in the circuit; we emphasize this fact this one time by placing several other current symbols about the circuit.

Our second step in the analysis is a choice of the voltage reference for each of the two resistors. The passive sign convention requires that the resistor cur rent and voltage variables be defined so that the current enters the terminal at which the positive voltage reference is located. Since we already (arbitrarily) selected the current direction, v_{R1} and v_{R2} are defined as in Fig. 3.12*c*.

The third step is the application of Kirchhoff's voltage law to the only closed path. Let us decide to move around the circuit in the clockwise direction, beginning at the lower left corner, and to write down directly every voltage first met at its positive reference, and to write down the negative of every voltage encountered at the negative terminal. Thus,

$$-v_{s1} + v_{R1} + v_{s2} + v_{R2} = 0 \quad [6]$$

We then apply Ohm's law to the resistive elements:

$$v_{R1} = R_1 i \text{ and } v_{R2} = R_2 i$$

Substituting into Eq. [6] yields

$$-v_{s1} + R_1 i + v_{s2} + R_2 i = 0$$

Since i is the only unknown, we find that

$$i = \frac{v_{s1} - v_{s2}}{R_1 + R_2}$$

The voltage or power associated with any element may now be obtained by applying $v = Ri$, $p = vi$, or $p = i^2R$.

PRACTICE •

3.5 In the circuit of Fig. 3.12*b*, $v_{s1} = 120$ V, $v_{s2} = 30$ V, $R_1 = 30$, and $R_2 = 15$. Compute the power absorbed by each element.

Ans: $p_{120V} = -240$ W; $p_{30V} = +60$ W; $p_{30} = 120$ W; $p_{15} = 60$ W.

Compute the power absorbed in each element for the circuit shown in Fig. 3.13*a*.

30
+ –
30
v
+ 30+ –
–
+
120 V –
$2v_A$
15
–
v_A +
i
$2v_A$
+
120 V
–
15
–
v_A +

(*a*) (*b*)

■ **FIGURE 3.13** (*a*) A single-loop circuit containing a dependent source. (*b*) The current *i* and voltage v_{30} are assigned.

(*Continued on next page*)

We first assign a reference direction for the current i and a reference polarity for the voltage v_{30} as shown in Fig. 3.13*b*. There is no need to assign a voltage to the 15 resistor, since the controlling voltage v_A for the dependent source is already available. (It is worth noting, however, that the reference signs for v_A are reversed from those we would have assigned based on the passive sign convention.)

This circuit contains a dependent voltage source, the value of which remains unknown until we determine v_A. However, its algebraic value $2v_A$ can be used in the same fashion as if a numerical value were available. Thus, applying KVL around the loop:

$$-120 + v_{30} + 2v_A - v_A = 0 \quad [7]$$

Using Ohm's law to introduce the known resistor values:

$$v_{30} = 30i \text{ and } v_A = -15i$$

Note that the negative sign is required since i flows into the negative terminal of v_A.

Substituting into Eq. [7] yields

$$-120 + 30i - 30i + 15i = 0$$

and so we find that

$$i = 8 \text{ A}$$

Computing the power *absorbed* by each element:

$$p_{120\text{V}} = (120)(-8) = -960 \text{ W}$$
$$p_{30} = (8)^2(30) = 1920 \text{ W}$$
$$p_{\text{dep}} = (2v_A)(8) = 2[(-15)(8)](8) = -1920 \text{ W}$$

$$p_{15} = (8)^2(15) = 960 \text{ W}$$

– +
12 V
8
30
+ v_x
7
–
$4v_x$
+
–

■ **FIGURE 3.14** A simple loop circuit.

PRACTICE •

3.6 In the circuit of Fig. 3.14, find the power absorbed by each of the five elements in the circuit.

Ans: (CW from left) 0.768 W, 1.92 W, 0.2048 W, 0.1792 W, −3.072 W.

In the preceding example and practice problem, we were asked to compute the power absorbed by each element of a circuit. It is difficult to think of a situation, however, in which *all* of the absorbed power quantities of a circuit would be positive, for the simple reason that the energy must come from somewhere. Thus, from simple conservation of energy, we expect that ***the sum of the absorbed power for each element of a circuit should be zero.*** In

other words, at least one of the quantities should be negative (neglecting the trivial case where the circuit is not operating). Stated another way, the sum of the supplied power for each element should be zero. More pragmatically,

the sum of the absorbed power equals the sum of the supplied power, which seems reasonable enough at face value.

Let's test this with the circuit of Fig. 3.13 from Example 3.5, which consists of two sources (one dependent and one independent) and two resistors. Adding the power absorbed by each element, we find

$$\sum_{\text{all elements}} p_{\text{absorbed}} = -960 + 1920 - 1920 + 960 = 0$$

In reality (our indication is the sign associated with the absorbed power) the 120 V source *supplies* +960 W, and the dependent source supplies +1920 W. Thus, the sources supply a total of 960 + 1920 = 2880 W. The resistors are expected to absorb positive power, which in this case sums to a total of 1920 + 960 = 2880 W. Thus, if we take into account each element of the circuit,

$$\sum p_{\text{absorbed}} = \sum p_{\text{supplied}}$$

as we expect.

Turning our attention to Practice Problem 3.6, the solution to which the reader might want to verify, we see that the absorbed powers sum to 0.768 + 1.92 + 0.2048 + 0.1792 − 3.072 = 0. Interestingly enough, the 12 V independent voltage source is absorbing +1.92 W, which means it is *dissipating* power, not supplying it. Instead, the dependent voltage source appears to be supplying all the power in this particular circuit. Is such a thing possible? We usually expect a source to supply positive power, but since we are employing idealized sources in our circuits, it is in fact possi ble to have a net power flow into any source. If the circuit is changed in some way, the same source might then be found to supply positive power. The result is not known until a circuit analysis has been completed.

3.5 THE SINGLE-NODE-PAIR CIRCUIT

The companion of the single-loop circuit discussed in Sec. 3.4 is the single node-pair circuit, in which any number of simple elements are connected between the same pair of nodes. An example of such a circuit is shown in Fig. 3.15*a*. KVL forces us to recognize that the voltage across each branch is the same as that across any other branch. *Elements in a circuit having a common voltage across them are said to be connected in* ***parallel.***

Find the voltage, current, and power associated with each element in the circuit of Fig. 3.15*a*.

We first define a voltage v and arbitrarily select its polarity as shown in Fig. 3.15*b*. Two currents, flowing in the resistors, are selected in con formance with the passive sign convention, as shown in Fig. 3.15*b*. *(Continued on next page)*

120 A R_1 30 A R_2 (*a*)

+

1/15 1/30 1/15 1/30

(*b*)

120 A R_1 30 A R_2 v _ i_1 i_2

■ **FIGURE 3.15** (*a*) A single-node-pair circuit. (*b*) A voltage and two currents are assigned.

Determining either current i_1 or i_2 will enable us to obtain a value for v. Thus, our next step is to apply KCL to either of the two nodes in the circuit. Equating the algebraic sum of the currents leaving the upper node to zero:

$$-120 + i_1 + 30 + i_2 = 0$$

Writing both currents in terms of the voltage v using Ohm's law

$$i_1 = 30v \text{ and } i_2 = 15v$$

we obtain

$$-120 + 30v + 30 + 15v = 0$$

Solving this equation for v results in

$$v = 2 \text{ V}$$

and invoking Ohm's law then gives

$$i_1 = 60 \text{ A and } i_2 = 30 \text{ A}$$

The absorbed power in each element can now be computed. In the two resistors,

$$p_{R1} = 30(2)^2 = 120 \text{ W and } p_{R2} = 15(2)^2 = 60 \text{ W}$$

and for the two sources,

$$p_{120\text{A}} = 120(-2) = -240 \text{ W and } p_{30\text{A}} = 30(2) = 60 \text{ W}$$

Since the 120 A source absorbs negative 240 W, it is actually *supplying* power to the other elements in the circuit. In a similar fashion, we find that the 30 A source is actually *absorbing* power rather than *supplying* it.

PRACTICE •

3.7 Determine v in the circuit of Fig. 3.16.

+

5 A 10 v 1 A 10 6 A

−

■ **FIGURE 3.16**

Ans: 50 V.

Determine the value of v and the power supplied by the independent current source in Fig. 3.17.

i_x

+

i_6

6 k $2i_x$ v 24 mA 2 k

−

■ **FIGURE 3.17** A voltage v and a current i_6 are assigned in a single-node-pair circuit containing a dependent source.

By KCL, the sum of the currents leaving the upper node must be zero, so that

$$i_6 - 2i_x - 0.024 - i_x = 0$$

Again, note that the value of the dependent source ($2i_x$) is treated the same as any other current would be, even though its exact value is not known until the circuit has been analyzed.

We next apply Ohm's law to each resistor:

$$i_6 = \frac{v}{6000} \quad \text{and} \quad i_x = \frac{-v}{2000}$$

Therefore,

$$\frac{v}{6000} - 2\left(\frac{-v}{2000}\right) - 0.024 - \left(\frac{-v}{2000}\right) = 0$$

and so $v = (600)(0.024) = 14.4$ V.

Any other information we may want to find for this circuit is now eas ily obtained, usually in a single step. For example, the power supplied by the independent source is $p_{24} = 14.4(0.024) = 0.3456$ W (345.6 mW).

PRACTICE •

3.8 For the single-node-pair circuit of Fig. 3.18, find i_A, i_B, and i_C.

+

i_B i_A i_C

5.6 A v_x 18 $0.1v_x$ 9 2 A

–

■ **FIGURE 3.18**

Ans: 3 A; −5.4 A; 6 A.

3.6 • SERIES AND PARALLEL CONNECTED SOURCES

It turns out that some of the equation writing that we have been doing for series and parallel circuits can be avoided by combining sources. Note, however, that all the current, voltage, and power relationships in the remainder of the circuit will be unchanged. For example, several voltage

+ –

+

v_1

v_2 + – $v_1 + v_2 - v_3$

=

v_3

–

– +

(*a*)

=

i_1 i_2 i_3 $i_1 - i_2 + i_3$ (*b*)

■ **FIGURE 3.19** (*a*) Series-connected voltage sources can be replaced by a single source. (*b*) Parallel current sources can be replaced by a single source.

sources in series may be replaced by an equivalent voltage source having a voltage equal to the algebraic sum of the individual sources (Fig. 3.19*a*). Parallel current sources may also be combined by algebraically adding the individual currents, and the order of the parallel elements may be rearranged as desired (Fig. 3.19*b*).

Determine the current *i* in the circuit of Fig. 3.20*a* by first combin ing the sources into a single equivalent voltage source.

To be able to combine the voltage sources, they must be in series. Since the same current (i) flows through each, this condition is satisfied. Starting from the bottom left-hand corner and proceeding clockwise,

$$-3 - 9 - 5 + 1 = -16 \text{ V}$$

so we may replace the four voltage sources with a single 16 V source having its negative reference as shown in Fig. 3.20*b*.
KVL combined with Ohm's law then yields

$$-16 + 100i + 220i = 0$$

or

$$i = \frac{16}{320} = 50 \text{ mA}$$

We should note that the circuit in Fig. 3.20*c* is also equivalent, a fact easily verified by computing i.

+
100
100
100

9 V

–

i i

i

\+

\+ – 16 V

– + 16 V

3 V

\+

–

– +

1 V

5 V

220

–

220 220

(*a*)

(*b*)

(*c*)

■ **FIGURE 3.20**

PRACTICE •

3.9 Determine the current *i* in the circuit of Fig. 3.21 after first replac ing the four sources with a single equivalent source.

47

i

\+ – 4 V

–

3 V

5 V

\+

–

\+

1 V

7

– +

■ **FIGURE 3.21**

Ans: −54 A.

Determine the voltage *v* in the circuit of Fig. 3.22*a* by first combin ing the sources into a single equivalent current source.

The sources may be combined if the same voltage appears across each one, which we can easily verify is the case. Thus, we create a new source, arrow pointing upward into the top node, by adding the currents that flow into that node:

$$2.5 - 2.5 - 3 = -3 \text{ A}$$

One equivalent circuit is shown in Fig. 3.22*b*.

KCL then allows us to write

$$-3 + \frac{v}{5} + \frac{v}{5} = 0$$

Solving, we find $v = 7.5$ V.

Another equivalent circuit is shown in Fig. 3.22*c*.

\+

5 *v* 5

2.5 A 2.5 A 3 A

–

(*a*)

+

5 v 5 −

(*b*)

+

5 v 5

3 A

−

(*c*)

−3 A

■ **FIGURE 3.22**

(*Continued on next page*)

PRACTICE •

3.10 Determine the voltage v in the circuit of Fig. 3.23 after first replacing the three sources with a single equivalent source.

+

10 v 10

5 A 1 A 6 A

−

■ **FIGURE 3.23**

Ans: 50 V.

To conclude the discussion of parallel and series source combinations, we should consider the parallel combination of two voltage sources and the series combination of two current sources. For instance, what is the equiva lent of a 5 V source in parallel with a 10 V source? By the definition of a voltage source, the voltage across the source cannot change; by Kirchhoff's voltage law, then, 5 equals 10 and we have hypothesized a physical impos sibility. Thus, *ideal* voltage sources in parallel are permissible only when each has the same terminal voltage at every instant. In a similar way, two current sources may not be placed in series unless each has the same cur rent, including sign, for every instant of time.

Determine which of the circuits of Fig. 3.24 are valid.

The circuit of Fig. 3.24*a* consists of two voltage sources in parallel. The value of each source is different, so this circuit violates KVL. For example, if a resistor is placed in parallel with the 5 V source, it is also in parallel with the 10 V source. The actual voltage across it is therefore ambiguous, and clearly the circuit cannot be constructed as indicated. If we attempt to build such a circuit in real life, we will find it impossible to locate "ideal" voltage sources—all real-world sources have an internal resistance. The presence of such resistance allows a voltage difference between the two *real*sources. Along these lines, the circuit of Fig. 3.24*b* is perfectly valid.

+ _ 5 V + _ 10 V (*a*)

R

1 A

+

(*b*)

(*c*)

2 V + _ 14 V –

R

1 A

FIGURE 3.24 (*a*) to (*c*) Examples of circuits with multiple sources, some of which violate Kirchhoff's laws.